Technical Addendum 2: The Consequences of Imposing a Non-negative Price Restriction

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Abstract

In this appendix we show that adopting a demand function of the form $p(q) = \max\{d - q, 0\}$ to avoid negative prices, leaves our main results and empirical predictions unaffected. The reason is that this change does not affect the expected value of information either across periods or destinations. This implies that the technical restriction (12), $\underline{d} > \frac{1}{2}E\mu$, adopted in the main text, while simplyifying the analysis considerably, is largely inconsequential. Therefore, to avoid the unnecessary technicalities displayed here, in the main text we impose it instead.

If we impose a restriction to avoid negative prices, the demand function takes the form $p(q) = \max\{d - q, 0\}$. In this appendix we show that adopting this natural restriction leaves our main results and empirical predictions unaffected. The main reason being that avoiding negative prices has no effect on the expected value of information either across periods or destinations. Intuitively, such a demand function "convexifies" the revenue function, providing implicit insurance to the risk neutral producer against the event of negative prices. Consequently, the producer is induced to take more risk, producing larger volumes conditional on entry, and becoming more propense to enter. [1]

To summarize, here we show as a result of forcing prices to be non-negative, optimal export quantities in $t = 1$ increase, while volumes in $t = 2$ remain unaffected. Since expected export profits also increase, there is also more entry. Because the surviving threshold in $t = 2$ remains unchanged ($\mu > \tau$), there is also more exit. Therefore our empirical predictions 2 and 3 are if anything, strengthened. Since optimal export quantities in $t = 1$ increase, while volumes in $t = 2$ remain unaffected, *predicted* average second year growth is lower, but still positive as long as minimum marginal costs lie above expected willigness to pay. Hence, also our empirical prediction 1 survives.

More entry and larger volumes in $t = 1$ translate into higher expected first period operational profits, inducing more experimentation. And because expected first period operational profits are larger, some firms that would have entered sequentially, now enter simultaneously, as well as some non-entrants

[1] Technically, it just introduces a first order stochastically dominant (FSD) shift in first period profitability, irrespective of destinations.

now will rather enter (sequentially) than not. Therefore our propositions 1 and 2 obtain, and so do their implications for trade policy (proposition 3). This is why in the main text we impose the (minor) technical restriction $\underline{d} > \frac{1}{2}E\mu$, instead of exposing the reader to the cumbersome technicalities displayed here.

Proposition 1 *First period export volumes are larger under a non-negative price restriction*

Proof. We want to show that:

$$q_1^{j*} \geq \widehat{q}_1^j$$

where:

$$\begin{aligned} q_1^{j*} &\in \arg\max_{q_1 \geq 0} E\left[\max\left\{\widetilde{d} - q_1, 0\right\} q_1 - (\widetilde{c} + \tau^j)q_1\right] \\ \widehat{q}_1^j &\in \arg\max_{q_1 \geq 0} E\left[\left(\widetilde{d} - q_1\right) q_1 - (\widetilde{c} + \tau^j)q_1\right] \end{aligned}$$

The corresponding necessary and sufficient FOCs are, under the assumption of independence between demand $(\widetilde{d})$ and supply $(\widetilde{c})$ shocks:

$$\begin{aligned} \underbrace{E\left\{-\mathbf{1}_{\{d>q_1^{j*}\}} q_1^{j*}\right\} + E\max\left\{\widetilde{d} - q_1^{j*}, 0\right\}}_{MR|p\geq 0} &= \underbrace{E\widetilde{c} + \tau^j}_{MC} \\ \underbrace{-\widehat{q}_1^j + \left(E\widetilde{d} - \widehat{q}_1^j\right)}_{MR} &= \underbrace{E\widetilde{c} + \tau^j}_{MC} \end{aligned}$$

Observing that $E\left\{-\mathbf{1}_{\{d>q\}}q\right\} = qE\left\{-\mathbf{1}_{\{d>q\}}\right\} = -q\left[1 - K(q)\right] \geq -q, \forall q \in \left[\underline{d}, \overline{d}\right]$, and that $E\max\left\{\widetilde{d} - q_1, 0\right\} \geq \max\left\{E\widetilde{d} - q_1, 0\right\} = \mathbf{1}_{\{Ed>q_1\}}\left(E\widetilde{d} - q_1\right) \geq \left(E\widetilde{d} - q_1\right)$, it follows that the marginal revenue is larger under the non-negative price restriction, while the marginal cost remains the same (MC) :

$$\left(MR|\, p \geq 0\right)(q_1) \geq MR(q_1), \forall q_1 \in \left[\underline{d}, \overline{d}\right]$$

Since the marginal revenue is a non-increasing function of the quantity[2], $q_1^{j*} \geq \widehat{q}_1^j$. ■

To be able to say if there is more or less (sequential) entry, we would need to know how do expected profits compare under the non-negative price restriction relative to its absence. First, notice that:

Proposition 2 *Conditional on entry, expected first period operational profits are larger when imposing a non-negative price restriction.*

[2] From Leibniz's rule, we have that $\frac{\partial(MR|p\geq 0)(q_1)}{\partial q_1} = -2(1 - K(q_1)) \geq -2 = \frac{\partial MR(q_1)}{\partial q_1}, \forall q_1$

Proof. Expected first period operational profits under a non-negative price restriction are:

$$\begin{aligned}
\Psi(q_1^{j*};\tau^j) - V(\tau^j) &= \left\{\max_{q_1\geq 0} E\left[\max\left\{\widetilde{d}-q_1,0\right\}q_1 - (\widetilde{c}+\tau^j)q_1\right]\right\} \\
&\geq \left\{\max_{q_1\geq 0}\left[\max\left\{E\widetilde{d}-q_1,0\right\}q_1 - (E\widetilde{c}+\tau^j)q_1\right]\right\} \\
&\geq \left\{\max_{q_1\geq 0}\left[\left(E\widetilde{d}-q_1\right)q_1 - (E\widetilde{c}+\tau^j)q_1\right]\right\} = \Psi(\widehat{q}_1^j;\tau^j) - V(\tau^j)
\end{aligned}$$

Where the second inequality follows from the convexity of the max operator and Jensen's inequality, and the third from noting that $\max\left\{E\widetilde{d}-q_1,0\right\} = \mathbf{1}_{\{Ed>q_1\}}\left(E\widetilde{d}-q_1\right) \geq \left(E\widetilde{d}-q_1\right), \forall q_1$.[3] ■

Second, it is also true that:

Corollary 3 *Operational profits under a non-negative price restriction are larger ([4])*

Proof. Notice that the definitions of $V(\tau^j)$ and of $W(\tau^B;F)$ in the main text remain unchanged by the imposition of a non-negative price-restriction. The reason being that they constitute the ex-ante evaluation of ex-post optimal entry decisions, which rule out negative prices, i.e. $\mu \geq \tau \Longrightarrow p^* \geq 0$:

$$\begin{aligned}
V(\tau^j) &= \int_{\tau^j}^{\overline{\mu}} \left(\frac{\mu^j-\tau^j}{2}\right)^2 dG(\mu) = E\left[\mathbf{1}_{\{\mu^j>\tau^j\}}\left(\frac{\mu^j-\tau^j}{2}\right)^2\right] \\
&= \Pr(\mu^j>\tau^j)E\left[\left(\frac{\mu^j-\tau^j}{2}\right)^2 \middle| \mu^j>\tau^j\right]; \\
W(\tau^B;F) &= \int_{\tau^B+2F^{\frac{1}{2}}}^{\overline{\mu}} \left[\left(\frac{\mu-\tau^B}{2}\right)^2 - F\right] dG(\mu) \\
&= \Pr(\mu>\tau^B+2F^{\frac{1}{2}})E\left[\left(\frac{\mu-\tau^B}{2}\right)^2 - F \middle| \mu>\tau^B+2F^{\frac{1}{2}}\right].
\end{aligned}$$

Therefore, the previous corollary implies that:

$$\Psi(q_1^{j*};\tau^j) \geq \Psi(\widehat{q}_1^j;\tau^j), \forall j$$

[3] After some tedious algebra, it can be shown that expected first period operational profits are equal to $\Psi(q_1^{j*};\tau^j) = \mathbb{P}(d>q_1^{j*})\left(q_1^{j*}\right)^2 + V(\tau^j)$.

[4] In the case of imperfect correlation across destinations, second period optimal output of sequential entrants is based on the conditional expectation of prices. As a result, prices can also be negative and the non-negative price restriction also constraints second period optimal outputs to be larger than they would absent the restriction. But because profits are larger, the new entry cutoff would also allow for more entry, and a similar reasoning applies.

■

As a result:

Corollary 4 *Both sequential and simultaneous entry strategies display higher profits under a non-negative price restriction. Therefore, the fixed cost entry thresholds under a non-negative price restriction,* F_*^{Sq} *and* F_*^{Sm}*, are less binding.*

Proof. Defining $\Psi(q_1^{j*};\tau^j) \equiv \Psi^*(\tau^j)$, $\Pi_*^{Sq} \equiv \Psi^*(\tau^A) + W(\tau^B;F) - F$, $\Pi_*^{Sm} \equiv \Psi^*(\tau^A) + \Psi^*(\tau^B) - 2F$, the previous corollary implies:

$$\Pi_*^{Sq} \geq \Pi^{Sq} \text{ and } \Pi_*^{Sm} \geq \Pi^{Sm}$$

Since the profit function is decreasing in the sunk entry cost F, we immediately have:

$$F_*^{Sq} \geq F^{Sq}$$

The definition of F_*^{Sm} and the previous corollary imply that:

$$F_*^{Sm} + W(\tau^B; F_*^{Sm}) = \Psi^*(\tau^B) \geq \Psi(\tau^B) = F^{Sm} + W(\tau^B; F^{Sm})$$

Since $\frac{d(F+W(\tau^B;F))}{dF} = G(\tau^B+2F^{\frac{1}{2}}) \geq 0$, we immediately have that $F_*^{Sm} \geq F^{Sm}$.

■

Firms that in the absence of a non-negative price restriction did not enter, now adopt a sequential entry strategy, and some of the previous sequential entrants, now would rather enter simultaneously. Therefore:

Corollary 5 $F_*^{Sq} > F_*^{Sm}$, *i.e. Proposition 1 survives a non-negative price restriction*

Proof.

$$F_*^{Sq} = \Psi^*(\tau^A)+W(\tau^B;F_*^{Sq}) > \Psi^*(\tau^A) \geq \Psi^*(\tau^B) > \Psi^*(\tau^B)-W(\tau^B;F_*^{Sm}) = F_*^{Sm}$$

where the weak inequality follows from the assumption that $\tau^A \leq \tau^B$, and the strict inequalities obtain because under perfect positive correlation, the option value of entering B sequentially is strictly positive, $W(\tau^B;F) > 0, \forall F$. ■

Consequently, our empirical predictions 2 (entry) and 3 (exit) prevail, and are even reinforced by the adoption of a non-negative price restriction. The next proposition shows that under an economically reasonable condition, also prediction 1 holds despite of being weakened:

Proposition 6 *Empirical prediction 1 holds if* $\underline{c} \geq Ed$*.*

Proof. From the FOC we obtain the following expression for q_1^{j*} :

$$q_1^{j*} = \mathbf{1}_{\{E\mu > \tau^j + \lambda\}} \frac{E\mu - (\tau^j + \lambda)}{2\mathbb{P}(d > q_1^{j*})}$$

where $\mathbb{P}(d > q_1^{j*}) \equiv \left[1 - K(q_1^{j*})\right] \leq 1$, and $\lambda \equiv \mathbb{P}(d \leq q_1^{j*}) E\left[d \middle| d \leq q_1^{j*}\right] \geq 0, \forall q_1^{j*} \in \left[\underline{d}, \overline{d}\right]$. We need to show that:

$$\underline{c} \geq Ed \Longrightarrow Eq_2^{j*} - q_1^{j*} \geq 0$$

Noting that $Eq_2^{j*} = E\widehat{q}_2^j = \frac{E\left[\mu | \mu > \tau^j\right] - \tau^j}{2}$, omitting the non-negativity restriction on quantities in the profit maximization problem, the above implication is equivalent to:

$$\underline{c} \geq Ed \Longrightarrow \frac{E\left[\mu \middle| \mu > \tau^j\right] - \tau^j}{2} \geq \frac{E\mu - (\tau^j + \lambda)}{2\mathbb{P}(d > q_1^{j*})}$$

The proof proceeds in 3 steps.

Step 1: Simplifying the RHS of the above implication.

After cancelling common terms and rearranging, we can express the RHS as :

$$\mathbb{P}(d > q_1^{j*}) E\left[\mu \middle| \mu > \tau^j\right] \geq E\mu - \mathbb{P}(d \leq q_1^{j*}) \left(E\left[d \middle| d \leq q_1^{j*}\right] + \tau^j\right)$$

by definition of λ. Since $E\mu = \mathbb{P}(d > q_1^{j*}) E\left[\mu \middle| d > q_1^{j*}\right] + \mathbb{P}(d \leq q_1^{j*}) E\left[\mu \middle| d \leq q_1^{j*}\right]$, plugging this expression into the above inequality and rearranging yields:

$$\mathbb{P}(d > q_1^{j*}) \left\{E\left[\mu \middle| \mu > \tau^j\right] - E\left[\mu \middle| d > q_1^{j*}\right]\right\} \geq \mathbb{P}(d \leq q_1^{j*}) \left\{E\left[\mu \middle| d \leq q_1^{j*}\right] - E\left[d \middle| d \leq q_1^{j*}\right] - \tau^j\right\}$$

Sustituting in the definition of $\widetilde{\mu} = \widetilde{d} - \widetilde{c}$, and taking advantage of the assumption of independence between demand and supply shocks, we get:

$$\mathbb{P}(d > q_1^{j*}) \left\{E\left[d \middle| d > c + \tau^j\right] - E\left[d \middle| d > q_1^{j*}\right] + Ec - E\left[c \middle| c < d - \tau^j\right]\right\} \geq \mathbb{P}(d \leq q_1^{j*}) \left\{-Ec - \tau^j\right\}$$

Noting that by the converse of Lemma 2 in the main text, $\mathbb{P}(d > q_1^{j*}) \left\{Ec - E\left[c \middle| c < d - \tau^j\right]\right\} \geq 0$. We can therefore move this term to the RHS of the inequality to obtain, after some simplifications:

$$\begin{aligned}\mathbb{P}(d &> q_1^{j*}) \left\{E\left[d \middle| d > c + \tau^j\right] - E\left[d \middle| d > q_1^{j*}\right]\right\} \geq \\ &\geq -\left\{Ec - E\left[c \middle| c < d - \tau^j\right]\right\} - \mathbb{P}(d \leq q_1^{j*}) \left\{E\left[c \middle| c < d - \tau^j\right] + \tau^j\right\}\end{aligned}$$

Therefore the RHS of the inequality is negative.

Step 2: The LHS of the inequality is positive if $c + \tau^j > q_1^{j*}, \forall c$.

It follows from an extension of Lemma 2 in the main text: [5]

$$\tau' \geq \tau \Longrightarrow E\left[\mu \mid \mu > \tau'\right] \geq E\left[\mu \mid \mu > \tau\right], \forall (\tau', \tau) \in (\underline{\mu}, \overline{\mu})$$

Step 3: $\underline{c} > Ed \Longrightarrow c + \tau^j > q_1^{j*}, \forall c$.
Notice that

$$c + \tau^j \geq \frac{c + \tau^j}{2\mathbb{P}(d > q_1^{j*})} \geq \frac{c + \tau^j - Ec - 2\tau^j}{2\mathbb{P}(d > q_1^{j*})} = \frac{c - Ec - \tau^j}{2\mathbb{P}(d > q_1^{j*})}$$

and also that

$$\frac{Ed - Ec - \tau^j}{2\mathbb{P}(d > q_1^{j*})} = \frac{E\mu - \tau^j}{2\mathbb{P}(d > q_1^{j*})} \geq \frac{E\mu - (\tau^j + \lambda)}{2\mathbb{P}(d > q_1^{j*})} = q_1^{j*}$$

Since the inequality must be true for all realizations of c, if $\underline{c} > Ed$ it must be true that $\frac{c - Ec - \tau^j}{2\mathbb{P}(d > q_1^{j*})} > \frac{Ed - Ec - \tau^j}{2\mathbb{P}(d > q_1^{j*})}$ and therefore that $\forall c, c + \tau^j > q_1^{j*}$, completing the proof. ■

[5] The proof proceeds as in lemma 2 in the main text: integrate by parts both expressions and subtract them to obtain

$$E\left[\mu \mid \mu > \tau'\right] - E\left[\mu \mid \mu > \tau\right] = \int_{\tau}^{\tau'} G(\mu \mid \mu > \tau) d\mu + \frac{G(\tau') - G(\tau)}{\left[1 - G(\tau')\right]\left[1 - G(\tau)\right]} \int_{\tau'}^{\overline{\mu}} \left[1 - G(\mu)\right] d\mu \geq 0$$

because $G(.)$ is a non-decreasing function.