Technical Addendum 1: Appendix B (complete)

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Abstract

This appendix is a complete version of the abridged appendix B in the main text. It provides all the technical proofs for the results as well as some details omitted there.

Here we show that our results generalize to the case of positive but imperfect statistical dependence between random variables μ^A and μ^B. In particular, we emphasize that the third-country result of Proposition 3 (parts a.2 and b.1) holds in the general case.

To keep the model symmetric, we assume distributions $G(\mu^A)$ and $G(\mu^B)$ are identical, although this is not essential. Upper-bar variables denote the counterparts to the variables in the main text under perfect correlation. For brevity, we denote $E\left[\mu^B \middle| \mu^A = u^A\right]$ by $E\left(\mu^B \middle| \mu^A\right)$, where u^A denotes a particular realization of the random variable μ^A.

Output choice: Output decisions in A at all times and in B at $t = 1$ are taken in the same way as in the main text. Output choice in B at $t = 2$ takes into account the realization of μ^A. From the convexity of the max function and Jensen's inequality,

$$\int_{\underline{\mu}^A}^{\overline{\mu}^A} \left[\max_{q^B \geq 0} \int_{\underline{\mu}^B}^{\overline{\mu}^B} (\mu^B - \tau^B - q^B) q^B dG(\mu^B | \mu^A)\right] dG(\mu^A) \geq \max_{q^B \geq 0} \int_{\underline{\mu}^B}^{\overline{\mu}^B} (\mu^B - \tau^B - q^B) q^B dG(\mu^B),$$

where $dG(\mu^B) = \int_{\underline{\mu}^A}^{\overline{\mu}^A} dG(\mu^B | \mu^A) dG(\mu^A)$. Expected profits are larger when an optimal production decision in B is made taking into account the experience acquired in A. By linearity of the expectation operator, optimal output is $\overline{q}_2^B(\tau^B) = \mathbf{1}_{\{E[\mu^B|\mu^A] > \tau^B\}} \left[\frac{E\left(\mu^B|\mu^A\right) - \tau^B}{2}\right]$.

Value of the sequential exporting strategy: The conditional expectation of random variable μ^B can be expressed as

$$E\left[\mu^B \middle| \mu^A\right] = E\mu^B + (u^A - E\mu^A) \underbrace{\int_{\underline{\mu}}^{\overline{\mu}} \left[-\frac{d}{du} G\left(w | \mu^A = u\right)\right]\Big|_{u=u_0} dw}_{\equiv \varpi}, \qquad (1)$$

where ϖ captures the statistical dependence between μ^A and μ^B.[1]

At $t = 2$ a firm enters market B if

$$\left(\frac{E\left[\mu^B \middle| \mu^A = u^A\right] - \tau^B}{2}\right)^2 \geq F \Leftrightarrow E\left(\mu^B \middle| \mu^A\right) \geq 2F^{1/2} + \tau^B. \qquad (2)$$

Define $\overline{F}_2^B(u^A;\tau^B)$ as the F that solves (2) with equality. The firm enters market B at $t = 2$ if $F \leq \overline{F}_2^B(u^A;\tau^B)$. Plugging (1) in (2) yields

$$\overline{F}_2^B(u^A;\tau^B) = \left(\frac{E\mu^B + \varpi(u^A - E\mu^A) - \tau^B}{2}\right)^2,$$

which is strictly decreasing in τ^B. Comparing $\overline{F}_2^B(u^A;\tau^B)$ with its analog under perfect correlation $F_2^B(\tau^B)$, defined on page 8, we have that $E\mu^A = E\mu^B$ implies $\lim_{\varpi \to 1} \overline{F}_2^B(u^A;\tau^B) = F_2^B(\tau^B)$.

Expressed in $t = 0$ expected terms, entering market B at $t = 2$ yields profits

$$\begin{aligned}
\overline{W}(\tau^B;F) &\equiv E_{\mu^A}\left[\max\left\{\max_{q^B \geq 0}\left[(E\left[\mu^B \middle| \mu^A\right] - \tau^B - q^B)q^B\right] - F, 0\right\}\right] \\
&= E_{\mu^A}\left\{\mathbf{1}_{\{\mu^A > \mu^{*A}(\varpi)\}}\left[\mathbf{1}_{\{E[\mu^B|\mu^A] > \tau^B\}}\left(\frac{E\left[\mu^B \middle| \mu^A\right] - \tau^B}{2}\right)^2 - F\right]\right\} \\
&= \int_{\mu^{*A}(\varpi)}^{\overline{\mu}}\left[\left(\frac{E\left(\mu^B \middle| \mu^A\right) - \tau^B}{2}\right)^2 - F\right] dG(\mu^A), \qquad (3)
\end{aligned}$$

where

$$\mu^{*A}(\varpi) \equiv \left(\frac{1}{\varpi}\right)(2F^{1/2} + \tau^B) - \left(\frac{1-\varpi}{\varpi}\right)E\mu^B$$

is the cutoff realization of export profitability in A above which a sequential exporter enters in B at $t = 2$.

For expositional clarity, notice that if μ^A and μ^B follow a bivariate normal distribution with parameters $(E\mu, E\mu, \sigma, \sigma, \rho)$, the cutoff varies with $\varpi = \rho$ as follows:

$$\frac{d\mu^{*A}(\rho)}{d\rho} = \frac{E\mu^B - (2F^{1/2} + \tau^B)}{\rho^2}.$$

Thus, when $E\mu^B > (2F^{1/2} + \tau^B)$ the cutoff rises as ρ increases, implying a lower value from experimentation. This simply reflects the fact that, if $E\mu^B > (2F^{1/2} + \tau^B)$, it is optimal to enter market B already at $t = 1$. Conversely, when $E\mu^B < (2F^{1/2} + \tau^B)$ the cutoff falls as ρ rises, implying a higher value from experimentation. This indicates that experimentation becomes more worthwhile as the statistical dependence between μ^A and μ^B increases. Experimentation is

[1] The proof of (1) can be found at the end of this appendix.

most valuable in the case of perfect correlation assumed in the main text, when it is worth $W(\tau^B; F)$. Experimentation is least valuable when μ^A and μ^B are independent, when it has no value.[2]

Choice of export strategy (extension of Proposition 1): As in the main text, $\overline{F}^{Sq}$ is the fixed cost that makes a firm indifferent between exporting sequentially and not exporting, whereas $\overline{F}^{Sm}$ makes a firm indifferent between simultaneous and sequential exporting strategies:

$$\overline{F}^{Sq} : \Psi(\tau^A) + \overline{W}(\tau^B; \overline{F}^{Sq}) = \overline{F}^{Sq}, \tag{4}$$
$$\overline{F}^{Sm} : \Psi(\tau^B) - \overline{W}(\tau^B; \overline{F}^{Sm}) = \overline{F}^{Sm}. \tag{5}$$

Since $\Psi(\tau^j)$ is monotonically decreasing in τ^j and $\tau^A \leq \tau^B$, and since $\overline{W}(\tau^B; F)$ is non-negative, there is a non-degenerate interval of fixed costs where firms choose the sequential export strategy.

Effects of trade liberalization (extension of Proposition 3): Differentiating $\overline{W}(\tau^B; F)$, we find

$$\frac{d\overline{W}(\tau^B;F)}{d\tau^B} = -\left\{ \int_{\mu^{*A}(\varpi)}^{\overline{\mu}} \left(\frac{E\left(\mu^B | \mu^A\right) - \tau^B}{2} \right) dG(\mu^A) + \right.$$
$$\left. + \frac{dG(\mu^{*A}(\varpi))}{\varpi} \underbrace{\left[\left(\frac{E\left(\mu^B \middle| \mu^{*A}(\varpi)\right) - \tau^B}{2} \right)^2 - F \right]}_{=0} \right\} < 0,$$

where the second term is zero –by construction of $\mu^{*A}(\varpi)$. Using this result and totally differentiating (4) and (5), we have that

$$\frac{d\overline{F}^{Sm}}{d\tau^A} = 0;$$
$$\frac{d\overline{F}^{Sm}}{d\tau^B} = \mathbf{1}_{\{E\mu > \tau^B\}} \left\{ \frac{-\left[\left(\frac{E\mu - \tau^B}{2} \right) + \int_{\tau^B}^{\overline{\mu}} \left(\frac{\mu - \tau^B}{2} \right) dG(\mu) - \int_{\mu^{*A}(\varpi)}^{\overline{\mu}} \left[\left(\frac{E\left(\mu^B | \mu^A\right) - \tau^B}{2} \right) \right] dG(\mu^A) \right]}{G(\mu^{*A}(\varpi))} \right\} \leq 0;$$
$$\frac{d\overline{F}^{Sq}}{d\tau^A} = -\frac{\mathbf{1}_{\{E\mu > \tau^A\}} \left(\frac{E\mu - \tau^A}{2} \right) - \int_{\tau^A}^{\overline{\mu}} \left(\frac{\mu - \tau^A}{2} \right) dG(\mu)}{2 - G(\mu^{*A}(\varpi))} < 0;$$
$$\frac{d\overline{F}^{Sq}}{d\tau^B} = -\frac{\int_{\mu^{*A}(\varpi)}^{\overline{\mu}} \left[\left(\frac{E\left(\mu^B | \mu^A\right) - \tau^B}{2} \right) \right] dG(\mu^A)}{2 - G(\mu^{*A}(\varpi))} < 0.$$

[2] Under independence between μ^A and μ^B, entry in A conveys no information about profitability in B. Thus, if it is not worthwhile to enter market B at $t = 2$, it is not worthwhile entering at $t = 1$ either. Conversely, if it pays to enter market B at $t = 2$, it must pay to enter also at $t = 1$, to avoid forgoing profits in the first period. Thus, under independence waiting to enter B at $t = 2$ is never optimal. For a formal proof of this statement, see F.N. 4 below.

The sign of all derivatives is non-positive, as in Lemma 1.[3] Therefore, the rest of the proof of parts a.2 and b.1. of Proposition 3 is straightforward, and proceeds analogously. The probability of sequential entry is qualitatively similar (considering the new entry cutoff $\mu^{*A}(\varpi)$). Exports vary at the intensive margin just as in the main text. Thus, trade liberalization has positive third-country effects also in the general case of positive statistical dependence between export profitability in A and B.

Comparing the general case with the polar cases: Here we show that when profitabilities are non-negatively regression dependent, the option value of learning one's export profitability in market B by entering in market A first, $\overline{W}(\tau^B;F)$, is bounded by the option values in the two polar cases of i.i.d. distributions (below) and perfect positive correlation (above).

We start with the lower bound. With i.i.d. marginal distributions of μ^A and μ^B we have $E\left(\mu^B\middle|\,\mu^A\right) = E\mu^B = E\mu$ and therefore $\varpi = 0$. Accordingly, the entry condition (2) becomes $E\mu \geq 2F^{1/2} + \tau^B$ so that

$$\lim_{\varpi\to 0}\overline{W}(\tau^B;F) = \mathbf{1}_{\{E\mu>2F^{1/2}+\tau^B\}}\left[\mathbf{1}_{\{E\mu>\tau^B\}}\left(\frac{E\mu-\tau^B}{2}\right)^2 - F\right].$$

But then entering market B sequentially is dominated by a simultaneous entry strategy at $t=1$: $\lim\limits_{\varpi\to 0}\overline{W}(\tau^B;F) < \Psi(\tau^B) - F$. The reason is that by entering at $t=2$ the firm only sacrifices positive expected profits, $V(\tau^B)$, because under independence, export experience in A is useless in B..Hence $\lim\limits_{\varpi\to 0}\overline{W}(\tau^B;F) = 0$, and the firm will never adopt a sequential entry strategy. Figure 1 illustrates this case.[4]

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[3] The sign of $\frac{d\overline{F}^{Sm}}{d\tau^B}$ when $E\mu > \tau^B$ depends on the sign of the numerator, which is negative whenever $\varpi = 1$ (perfect correlation), as shown in the main text. Property (3) at the end of this appendix proves that $\left|\frac{\partial W(\tau^B;F)}{\partial\tau^B}\right| \geq \left|\frac{\partial\overline{W}(\tau^B;F)}{\partial\tau^B}\right|, \forall\varpi \geq 0$. Consequently, the numerator will even more so remain negative for any other degree of non-negative statistical dependence. Therefore $\frac{d\overline{F}^{Sm}}{d\tau^B} < 0$.

[4] Analytically, we only need to examine whether there are values of F such that $\Pi^{Sm} \leq \overline{\Pi}^{Sq}$ when $\varpi = 0$:

$$\Psi(\tau^A) + \Psi(\tau^B) - 2F \leq \Psi(\tau^A) + \lim_{\varpi\to 0}\overline{W}(\tau^B;F) - F$$

Cancelling terms and substituting the expression for $\lim\limits_{\varpi\to 0}\overline{W}(\tau^B;F)$,

$$\Psi(\tau^B) - F \leq \mathbf{1}_{\{E\mu>2F^{1/2}+\tau^B\}}\left[\mathbf{1}_{\{E\mu>\tau^B\}}\left(\frac{E\mu-\tau^B}{2}\right)^2 - F\right]$$

According to the first indicator function, we must distinguish two cases: (i) if $E\mu > 2F^{1/2} + \tau^B$, the inequality reduces to $V(\tau^B) \leq 0$, which is false. Hence, there is no value of F that satisfies it. (ii) If $E\mu \leq 2F^{1/2} + \tau^B$, the inequality reduces to $\Psi(\tau^B) - F \leq 0$, meaning that the only values of F that satisfy the inequality are those for which early entry in B is not worth ($e_1^B = 0$). Since late entry in B is worth only when $\Psi(\tau^B) - V(\tau^B) \geq F$, $V(\tau^B) > 0$ and the above inequality imply that:

$$F \geq \Psi(\tau^B) > \Psi(\tau^B) - V(\tau^B) \geq F,$$

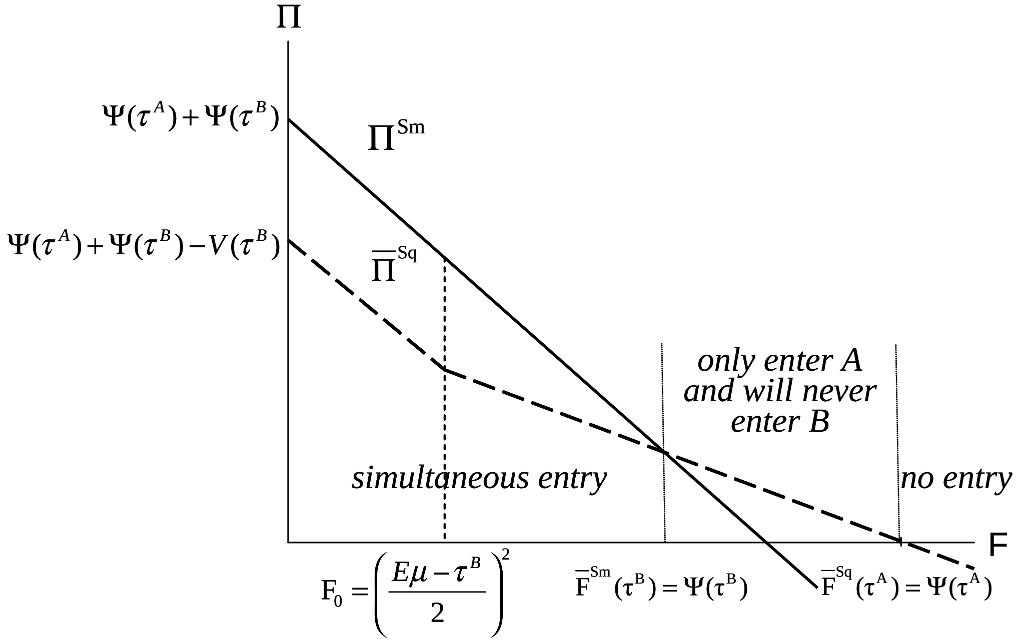

Figure 1: With independent export profitabilities ($\varpi = 0$), a firm will never enter sequentially.

Consider now the upper bound. Under perfect positive correlation between μ^A and μ^B, the term that captures the degree of statistical dependence ϖ in expression (1) becomes [5]:

$$\int_{\underline{\mu}}^{\overline{\mu}} \left[-\frac{d}{du} G\left(w | \, \mu^A = u \right) \right] \Big|_{u=u_0} dw = 1.$$

Plugging this condition into expression (3), and since $E\mu^B = E\mu^A = E\mu$, $E\left(\mu^B \middle| \, \mu^A \right) = \mu^A$, we obtain that as $\varpi \to 1$:

$$\lim_{\varpi \to 1} \overline{W}(\tau^B; F) = W(\tau^B; F)$$

Finally, notice that:

$$\begin{aligned} W(\tau^B; F) &= E_{\mu^B} \left[\max \left\{ \max_{q^B \geq 0} (\mu^B - \tau^B - q^B) q^B - F, 0 \right\} \right] \\ &= E_{\mu^A} \left[E_{\mu^B|\mu^A} \left(\max \left\{ \max_{q^B \geq 0} (\mu^B - \tau^B - q^B) q^B - F, 0 \right\} \middle| \, \mu^A \right) \right] \\ &\geq E_{\mu^A} \left[\max \left\{ \max_{q^B \geq 0} E_{\mu^B|\mu^A} \left[(\mu^B - \tau^B - q^B) q^B \middle| \, \mu^A \right] - F, 0 \right\} \right] \\ &= E_{\mu^A} \left[\max \left\{ \max_{q^B \geq 0} \left[(E\left[\mu^B \middle| \, \mu^A \right] - \tau^B - q^B) q^B \right] - F, 0 \right\} \right] \\ &= \overline{W}(\tau^B; F), \forall \varpi \geq 0 \end{aligned}$$

where the inequality obtains from applying twice Jensen's inequality and the convexity of the max $\{.\}$ operator, while the third equality above follows from the law of iterated expectations, i.e. $E_{\mu^B}\left[f(\mu^B) \right] = E_{\mu^A}\left[E_{\mu^B|\mu^A}\left(f(\mu^B) \middle| \, \mu^A \right) \right]$. Therefore:

$$0 \leq \overline{W}(\tau^B; F) \leq W(\tau^B; F)$$

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a contradiction. Therefore, there is no value of F either that satisfies the inequality. Consequently, the sequential entry strategy is never adopted.

[5] Under perfect positive correlation between μ^A and μ^B,

$$G\left(w | \, \mu^A = u \right) = \begin{cases} 1 & \text{if } w \geq u \\ 0 & \text{if } w < u, \end{cases}$$

which is a Heavyside step function (or unit step function) $T(w-u) = \int_{\underline{\mu}}^{u} \delta(w-s) ds$, where $\delta(w-s)$ denotes a Dirac delta function $\delta(w-s) = \begin{cases} +\infty & \text{if } w = s \\ 0 & \text{otherwise} \end{cases}$ such that $\int_{\underline{\mu}}^{\overline{\mu}} \delta(w - s) dw = 1, \forall s \left[\underline{\mu}, \overline{\mu} \right]$. Since $\frac{d}{du} T(w-u) = -\delta(w-u)$ we have:

$$\int_{\underline{\mu}}^{\overline{\mu}} \left[-\frac{d}{du} G\left(w | \, \mu^A = u \right) \right] \Big|_{u=u_0} dw = \int_{\underline{\mu}}^{\overline{\mu}} \delta(w - u_0) dw = 1.$$

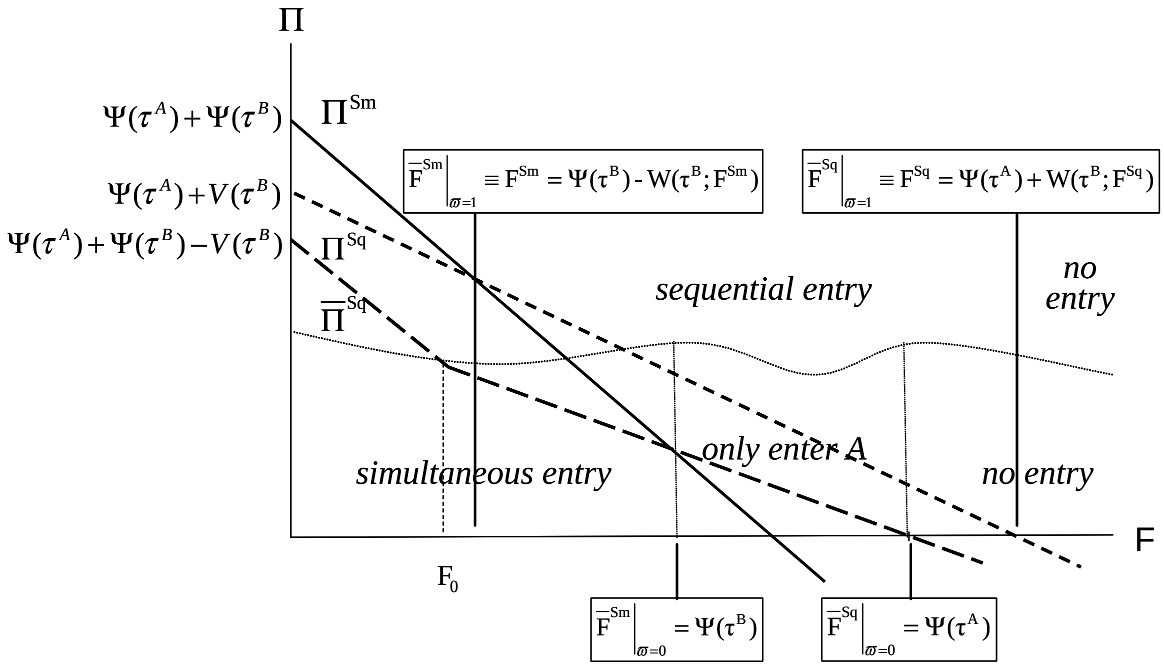

Figure 2: Bounds on sunk entry thresholds, F^{Sm} and F^{Sq}, as a function of the statistical dependence (ϖ) between export profitabilites.

As in the main text, those bounds on the option values correspond to sunk entry cost thresholds above which the exporter prefers to enter sequentially (F^{Sm}), as illustrated in Figure 2. [6]Hence, the region defined by Proposition 1 where it is optimal to adopt a sequential entry strategy shrinks as the statistical dependence of export profitabilities across the two destinations is reduced from

[6]Notice that in figure (in accordance with notation in the main text) $\Pi^{Sq} \equiv \overline{\Pi}^{Sq}\Big|_{\varpi=1}$ whereas, $\underline{\Pi}^{Sq} \equiv \overline{\Pi}^{Sq}\Big|_{\varpi=0}$. Also notice from the figure that $\Pi^{Sq}(F) > \underline{\Pi}^{Sq}(F), \forall F \leq \overline{F}^{Sq}\Big|_{\varpi=1}$. The only non-trivial point is to prove that $\Pi^{Sq}(0) = V(\tau^B) \geq \underline{\Pi}^{Sq}(0) = \Psi(\tau^B) - V(\tau^B)$ which follows from the application of Jensen's inequality and the convexity of the $\max\{.\}$ operator:

$$\begin{aligned} V(\tau^B) &= E\left[\max_{q\geq 0}(\widetilde{\mu} - \tau^B - q)q\right] = E\left[\mathbf{1}_{\{\mu>\tau^B\}}\left(\frac{\mu-\tau^B}{2}\right)^2\right] \\ &\geq \max_{q\geq 0} E\left[(\widetilde{\mu} - \tau^B - q)q\right] = \mathbf{1}_{\{E\mu>\tau^B\}}\left(\frac{E\mu-\tau^B}{2}\right)^2 \equiv \Psi(\tau^B) - V(\tau^B) \end{aligned}$$

perfect to no correlation:

$$\begin{aligned}
F^{Sq} - F^{Sm} &\equiv \Psi(\tau^A) + W(\tau^B; F^{Sq}) - \left[\Psi(\tau^B) - W(\tau^B; F^{Sm})\right] \\
&= \Psi(\tau^A) - \Psi(\tau^B) + W(\tau^B; F^{Sq}) + W(\tau^B; F^{Sm}) \\
&\geq \Psi(\tau^A) - \Psi(\tau^B) + \overline{W}(\tau^B; \overline{F}^{Sq}) + \overline{W}(\tau^B; \overline{F}^{Sm}) \\
&\equiv \left.\overline{F}^{Sq} - \overline{F}^{Sm}\right|_{1>\varpi>0} \\
&\geq \Psi(\tau^A) - \Psi(\tau^B) \\
&\equiv \left.\overline{F}^{Sq} - \overline{F}^{Sm}\right|_{\varpi=0}
\end{aligned}$$

Derivation of (1)**:** Here we show how the conditional expectation can be expressed as a function of the unconditional expectation, as in (1). Integrating by parts both expectations and taking the difference we obtain:

$$\begin{aligned}
E\left[\mu^B \middle| \mu^A = u^A\right] - E\left[\mu^B\right] &= \int_{\underline{\mu}}^{\overline{\mu}} \left[G_B(w) - G\left(w \middle| \mu^A = u^A\right)\right] dw \\
&= \int_{\underline{\mu}}^{\overline{\mu}} \left[G\left(w \middle| \mu^A \leq \overline{\mu}\right) - G\left(w \middle| \mu^A = u^A\right)\right] dw
\end{aligned}$$

Since $G_B(w) \equiv G\left(\mu^B \leq w, \mu^A \leq \overline{\mu}\right) = G\left(\mu^B \leq w \middle| \mu^A \leq \overline{\mu}\right) G_A(\mu^A \leq \overline{\mu}) = G\left(\mu^B \leq w \middle| \mu^A \leq \overline{\mu}\right)$, $\forall w \in \left[\underline{\mu}, \overline{\mu}\right]$, because $G_A(\mu^A \leq \overline{\mu}) = 1$. By definition, $G\left(w \middle| \mu^A \leq \overline{\mu}\right) = \int_{\underline{\mu}}^{\overline{\mu}} G\left(w \middle| \mu^A = u\right) dG_A(u)$, which inserted above yields:

$$\begin{aligned}
E\left[\mu^B \middle| \mu^A = u^A\right] - E\left[\mu^B\right] &= \int_{\underline{\mu}}^{\overline{\mu}} \left[\int_{\underline{\mu}}^{\overline{\mu}} G\left(w \middle| \mu^A = u\right) dG_A(u) - G\left(w \middle| \mu^A = u^A\right)\right] dw \\
&= \int_{\underline{\mu}}^{\overline{\mu}} \left[\int_{\underline{\mu}}^{\overline{\mu}} G\left(w \middle| \mu^A = u\right) dG_A(u) - G\left(w \middle| \mu^A = u^A\right) \underbrace{\int_{\underline{\mu}}^{\overline{\mu}} dG_A(u)}_{=1}\right] dw \\
&= \int_{\underline{\mu}}^{\overline{\mu}} \int_{\underline{\mu}}^{\overline{\mu}} \left[G\left(w \middle| \mu^A = u\right) - G\left(w \middle| \mu^A = u^A\right)\right] dG_A(u)\, dw.
\end{aligned}$$

Now assuming that $G(w|.) \in C^1\left[\underline{\mu}, \overline{\mu}\right]$, by the mean-value theorem,

$$\exists u_0 \in \left[\underline{\mu}, \overline{\mu}\right] : G\left(w \middle| \mu^A = u\right) - G\left(w \middle| \mu^A = u^A\right) = (u - u^A)\left(\left.\left[\frac{d}{du} G\left(w \middle| \mu^A = u\right)\right]\right|_{u=u_0}\right)$$

we obtain:

$$E\left[\mu^B \middle| \mu^A = u^A\right] - E\left[\mu^B\right] = \int_{\underline{\mu}}^{\overline{\mu}} \int_{\underline{\mu}}^{\overline{\mu}} \left[(u - u^A)\left(\left.\left[\frac{d}{du} G\left(w \middle| \mu^A = u\right)\right]\right|_{u=u_0}\right)\right] dG_A(u)\, dw$$

Since the term $\left[\frac{d}{du}G\left(w|\,\mu^A=u\right)\right]\big|_{u=u_0}$ is a constant, it follows that:

$$\begin{aligned} E\left[\mu^B\middle|\,\mu^A=u^A\right]-E\left[\mu^B\right] &= \left(E\left[\mu^A\right]-u^A\right)\int_{\underline{\mu}}^{\overline{\mu}}\left[\left[\frac{d}{du}G\left(w|\,\mu^A=u\right)\right]\Big|_{u=u_0}\right]dw \\ &= \left(u^A-E\left[\mu^A\right]\right)\int_{\underline{\mu}}^{\overline{\mu}}\left(-\left[\frac{d}{du}G\left(w|\,\mu^A=u\right)\right]\Big|_{u=u_0}\right)dw \end{aligned}$$

We use Lehmann's (1966, p.1143-4) definition of *regression dependence*, which is in our context:

Definition 1 *μ^B is positively (negatively) regression dependent on μ^A if $G\left(\mu^B\leq w|\,\mu^A=u\right)$ is non-increasing (non-decreasing) in u.*

Our assumption of statistical dependence between μ^A and μ^B implies regression dependence. Thus we can sign the integrand in the last equality above. Finally by rearranging the last equality, we obtain (1): if μ^B and μ^A are positively associated, $\left[\frac{d}{du}G\left(w|\,\mu^A=u\right)\right]\big|_{u=u_0}\leq 0$ and $\left(-\left[\frac{d}{du}G\left(w|\,\mu^A=u\right)\right]\big|_{u=u_0}\right)\geq 0,\forall w$ so that $\int_{\underline{\mu}}^{\overline{\mu}}\left(-\left[\frac{d}{du}G\left(w|\,\mu^A=u\right)\right]\big|_{u=u_0}\right)dw\geq 0$. Now if export profitability in A was better than expected ($u^A\geq E\left[\mu^A\right]$), expected export profitability to B increases ($E\left[\mu^B\middle|\,\mu^A=u^A\right]\geq E\left[\mu^B\right]$).

Example: normal distribution. Consider a joint normal distribution of μ^A and μ^B. It is enough to compute[7]:

$$\int_{-\infty}^{+\infty}\left[-\frac{d}{du}G\left(w|\,\mu^A=u\right)\right]\Big|_{u=u_0}dw$$

where

$$G\left(w|\,\mu^A=u\right)=\int_{-\infty}^{w}\frac{1}{\sigma_B\sqrt{2\pi}\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{s-(E\mu^B+\rho\frac{\sigma_B}{\sigma_A}(u-E\mu^A))}{\sigma_B}\right]^2\right\}ds$$

is the conditional distribution of μ^B, such that $(\mu^B|\,\mu^A=u)\sim N(E\mu^B+\rho\frac{\sigma_B}{\sigma_A}(u-E\mu^A),\sigma_B^2(1-\rho^2))$. We note that[8]: (i) $dG\left(s|\,\mu^A=u\right)$ is a continuous function of $(s,u)\in\mathbb{R}^2$, (ii) $\frac{d}{du}[dG\left(s|\,\mu^A=u\right)]$ exists and is continuous, and (iii) $\int_{-\infty}^{w}dG\left(s|\,\mu^A=u\right)ds$ is continuous. Therefore we can differentiate inside the

[7] Although expression (1) is defined for random variables on bounded supports, we conjecture that it can be extended to random variables over unbounded supports as long as their c.d.f., say $G(\bullet)$, possess an absolute moment of order $\psi>0$, i.e if and only if $|\mu|^{\psi-1}\left[1-G(\mu)+G(-\mu)\right]$ is integrable over $(-\infty,+\infty)$, (see Lemma 2 in Feller (1966, p.149).

[8] Facts (i) - (iii) are stated without proof, but since $\exp(-\frac{x^2}{2})$ is continuous, positive and bounded above by an integrable function ($\exp(-|x|+1):\int_{\mathbb{R}}\exp(-|x|+1)dx=2e$), on $\mathbb{R}$, the proofs are left to the interested reader.

integral:

$$\begin{aligned}
\frac{d}{du}G\left(w|\,\mu^A=u\right) &= \int_{-\infty}^{w}\frac{d}{du}\left[dG\left(s|\,\mu^A=u\right)\right]ds \\
&= \int_{-\infty}^{w}\Bigg(\frac{1}{\sigma_B\sqrt{2\pi}\sqrt{1-\rho^2}}\left[\frac{\rho\frac{\sigma_B}{\sigma_A}}{\sigma_B(1-\rho^2)}\left[\frac{s-(E\mu^B+\rho\frac{\sigma_B}{\sigma_A}(u-E\mu^A))}{\sigma_B}\right]\right]\times \\
&\quad \times\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{s-(E\mu^B+\rho\frac{\sigma_B}{\sigma_A}(u-E\mu^A))}{\sigma_B}\right]^2\right\}\Bigg)ds \\
&= -\rho\frac{\sigma_B}{\sigma_A}G\left(w|\,\mu^A=u\right),
\end{aligned}$$

which substituted above yields:

$$\int_{-\infty}^{+\infty}\left[-\frac{d}{du}G\left(w|\,\mu^A=u\right)\right]\Bigg|_{u=u_0}dw=\int_{-\infty}^{+\infty}\rho\frac{\sigma_B}{\sigma_A}G\left(w|\,\mu^A=u_0\right)dw=\rho\frac{\sigma_B}{\sigma_A}$$

This yields the well-known relationship:

$$E\left[\mu^B\middle|\,\mu^A\right]=E\left[\mu^B\right]+\rho\frac{\sigma_B}{\sigma_A}\left[\mu^A-E\left[\mu^A\right]\right] \tag{6}$$

which is a particular case of (1) where $\varpi\equiv\rho\frac{\sigma_B}{\sigma_A}$.

Formal definition and some properties of the option value functions: The formal definition of the option value functions are: ([9])

$$
\begin{aligned}
V(\tau^j) &= E\left[\max_{q^j\geq 0}(\mu^j-\tau^j-q^j)q^j\right] = E\left[\mathbf{1}_{\{\mu^j>\tau^j\}}\left(\frac{\mu^j-\tau^j}{2}\right)^2\right] \\
&= \Pr(\mu^j>\tau^j)E\left[\left(\frac{\mu^j-\tau^j}{2}\right)^2\middle|\,\mu^j>\tau^j\right] \\
&= \int_{\tau^j}^{\overline{\mu}}\left(\frac{\mu^j-\tau^j}{2}\right)^2 dG(\mu); \\
W(\tau^B;F) &= E\left[\max\left\{\max_{q^B\geq 0}(\mu-\tau^B-q^B)q^B-F,0\right\}\right] \\
&= E\left\{\mathbf{1}_{\left\{\mu>\tau^B+2F^{\frac{1}{2}}\right\}}\left[\mathbf{1}_{\{\mu>\tau^B\}}\left(\frac{\mu-\tau^B}{2}\right)^2-F\right]\right\} \\
&= \Pr(\mu>\tau^B+2F^{\frac{1}{2}})E\left[\left(\frac{\mu-\tau^B}{2}\right)^2-F\,\middle|\,\mu>\tau^B+2F^{\frac{1}{2}}\right] \\
&= \int_{\tau^B+2F^{\frac{1}{2}}}^{\overline{\mu}}\left[\left(\frac{\mu-\tau^B}{2}\right)^2-F\right]dG(\mu)
\end{aligned}
$$

Some properties:

(1) *Expected profits are larger when optimal decisions are taken on the basis of more information.*

From these expressions, by the convexity of the max $\{.\}$ operator and Jensen's inequality we obtain:

$$
\begin{aligned}
V(\tau^j) &\equiv E\left[\max_{q^j\geq 0}(\mu^j-\tau^j-q^j)q^j\right] = E\left[\left(\widehat{q}_2^j\right)^2\right] \\
&\geq \max_{q^j\geq 0}E\left[(\mu^j-\tau^j-q^j)q^j\right] = \left(\widehat{q}_1^j\right)^2 \equiv \Psi(\tau^j)-V(\tau^j) \\
W(\tau^B;F) &\geq \overline{W}(\tau^B;F), \forall\varpi\geq 0
\end{aligned}
$$

[9] Although in the main text we have adopted the simplest analytic expressions, sometimes doing so obscures the implicit timing behind them. As an example, the value of early entry in B can be expressed as:

$$
\begin{aligned}
\max\left\{\Psi(\tau^B)-F,0\right\} &= \max\left\{\begin{array}{c}\mathbf{1}_{\{E\mu^B>\tau^B\}}\left(\frac{E\mu^B-\tau^B}{2}\right)^2+\mathbf{1}_{\{E\mu^B\leq\tau^B\}}\left(E\mu^B-\tau^B-\varepsilon\right)\varepsilon+ \\ +\mathbf{1}_{\{\widehat{q}_1^B>0\}}E\left[\mathbf{1}_{\{\mu^B>\tau^B\}}\left(\frac{\mu^B-\tau^B}{2}\right)^2\right]-F,0\end{array}\right\} \\
&= \mathbf{1}_{\{e_1^B=1\}}\left(\begin{array}{c}\mathbf{1}_{\{E\mu^B>\tau^B\}}\left(\frac{E\mu^B-\tau^B}{2}\right)^2+\mathbf{1}_{\{E\mu^B\leq\tau^B\}}\left(E\mu^B-\tau^B-\varepsilon\right)\varepsilon+ \\ +\mathbf{1}_{\{\widehat{q}_1^B>0\}}E\left[\mathbf{1}_{\{\mu^B>\tau^B\}}\left(\frac{\mu^B-\tau^B}{2}\right)^2\right]-F\end{array}\right)
\end{aligned}
$$

where the second inequality has already been established above.

(2) *The smaller the degree of spatial correlation across destinations, the less would firms find optimal a sequential entry strategy.*

From the main text, we also know that:

$$V(\tau^B) \geq W(\tau^B; F)$$

and therefore:

$$V(\tau^B) \geq W(\tau^B; F) \geq \overline{W}(\tau^B; F), \forall \varpi \geq 0$$

meaning that the option value of early entry is larger than the option value of late entry (only because sunk export costs have already been incurred by early entrants, but not by late ones), and the more so, the less informative entering A is about export success in B.

(3) *The impact of a reduction in trade barriers on the option value of a sequential entry strategy, increases with the degree of spatial correlation across destinations.*

$$\left| \frac{\partial W(\tau^B; F)}{\partial \tau^B} \right| \geq \left| \frac{\partial \overline{W}(\tau^B; F)}{\partial \tau^B} \right|, \forall \varpi \geq 0$$

Computing:

$$\begin{aligned}
\frac{\partial W(\tau^B; F)}{\partial \tau^B} &= \frac{\partial}{\partial \tau^B} \left(E_{\mu^B} \left[\max \left\{ \max_{q^B \geq 0} (\mu^B - \tau^B - q^B) q^B - F, 0 \right\} \right] \right) \\
&= -E_{\mu^B} \left[\mathbf{1}_{\left\{ \mu^B > \tau^B + 2F^{\frac{1}{2}} \right\}} \left(\frac{\mu^B - \tau^B}{2} \right) \right] \\
&= -E_{\mu^B} \left[\max \left\{ \left[\max_{q^B \geq 0} (\mu^B - \tau^B - q^B) q^B \right]^{\frac{1}{2}} - F^{\frac{1}{2}}, 0 \right\} \right] \\
&= -E_{\mu^A} \left[E_{\mu^B | \mu^A} \left(\max \left\{ \left[\max_{q^B \geq 0} (\mu^B - \tau^B - q^B) q^B \right]^{\frac{1}{2}} - F^{\frac{1}{2}}, 0 \right\} \middle| \mu^A \right) \right] \\
&\leq -E_{\mu^A} \left[\max \left\{ E_{\mu^B | \mu^A} \left(\left[\max_{q^B \geq 0} (\mu^B - \tau^B - q^B) q^B \right]^{\frac{1}{2}} \middle| \mu^A \right) - F^{\frac{1}{2}}, 0 \right\} \right] \\
&\leq -E_{\mu^A} \left[\max \left\{ \left[\max_{q^B \geq 0} \left[(E \left[\mu^B \middle| \mu^A \right] - \tau^B - q^B) q^B \right] \right]^{\frac{1}{2}} - F^{\frac{1}{2}}, 0 \right\} \right] \\
&= -E_{\mu^A} \left[\mathbf{1}_{\left\{ E[\mu^B | \mu^A] > \tau^B + 2F^{\frac{1}{2}} \right\}} \left(\frac{E \left[\mu^B \middle| \mu^A \right] - \tau^B}{2} \right) \right] \\
&= -E_{\mu^A} \left[\mathbf{1}_{\{ \mu^A > \mu^{*A}(\varpi) \}} \left(\frac{E \left[\mu^B \middle| \mu^A \right] - \tau^B}{2} \right) \right] \\
&= \frac{\partial}{\partial \tau^B} \left(E_{\mu^A} \left[\max \left\{ \max_{q^B \geq 0} \left[(E \left[\mu^B \middle| \mu^A \right] - \tau^B - q^B) q^B \right] - F, 0 \right\} \right] \right) \\
&\equiv \frac{\partial \overline{W}(\tau^B; F)}{\partial \tau^B}
\end{aligned}$$

Since taking absolute values, reverses the inequalities, the proof is complete. The fourth equality applies the law of iterated expectations, while the first inequality follows from the convexity of the max $\{.\}$ operator and Jensen's inequality, and the negative sign. The second follows from noting that:

$$
\begin{aligned}
E_{\mu^B|\mu^A}\left(\left[\max_{q^B\geq 0}(\mu^B-\tau^B-q^B)q^B\right]^{\frac{1}{2}}\middle|\mu^A\right) &= E_{\mu^B|\mu^A}\left(\mathbf{1}_{\{\mu^B>\tau^B\}}\left[\frac{\mu^B-\tau^B}{2}\right]\middle|\mu^A\right)\\
&\geq E_{\mu^B|\mu^A}\left(\frac{\mu^B-\tau^B}{2}\middle|\mu^A\right)\\
&= \left[\max_{q^B\geq 0}\left[(E\left[\mu^B\middle|\mu^A\right]-\tau^B-q^B)q^B\right]\right]^{\frac{1}{2}}
\end{aligned}
$$

Then, subtract $-F^{\frac{1}{2}}$ and apply the max $\{.,0\}$ operator on both sides of the inequality, take expectations wrt. μ^A and switch signs to reverse the direction of the inequality.

References

Feller, W. (1966), *An Introduction to Probability Theory and Its Applications*, Vol. II, Wiley Series in Probability and Mathematical Statistics, John Wiley and Sons.