### Lecture 9: The Gravity Equation

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### A brief history of gravity

• Tinbergen (1962) : empirically successful relationship

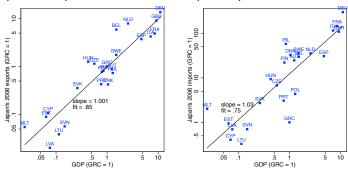
$$X_{ij} = G(Y_i)^a (Y_j)^b (d_{ij})^c$$

but dismissed for its lack of theoretical underpinnings

- Mid-90s: 'admission' of the gravity equation
  - ► Trefler (1995) : "Missing trade" in HOV, importance of trade costs
  - ▶ McCallum (1995) : "Border effect" estimated in a gravity context
- 2000's: micro-fundations for the gravity equation
   Eaton & Kortum (2002), Anderson & van Wincoop (2003), Chaney (2008), Melitz & Ottaviano (2008)
- Nowadays, gravity is a central component of trade theories (see eg Arkolakis et al, 2012)

#### Trade and the size of countries

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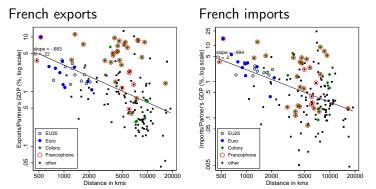


Correlation between the Japan-EU trade and the size of partners. The x-axis measure the GDP of each EU member relative to Greek GDP. The y-axis measure the size of Japanese exports in each country (left-hand side) and the volume of Japanese imports from each country (right-hand side), both relative to Greece. Data are for 2006. Source: Head & Mayer (2014).

#### GDP elasticities around 1



### Trade and distance



Correlation between the volume of trade and the distance between partners. The x-axis is the distance from France, expressed in kilometers. The x-axis measures the size of French exports (left-hand side) and the size of French imports (right-hand side), both expressed in relative terms with respect of the destination country's GDP. Data are for 2006. Source: Head & Mayer (2014).

### **Definitions**

- A model of bilateral interactions in which size and distance effects enter multiplicatively (analogy to Newton's gravity)
- General gravity model :

$$X_{ij} = GS_i M_j \phi_{ij}$$

 $S_i$  captures i's "capabilities" as a supplier,  $M_i$  captures i's characteristics that promote imports,  $0 \le \phi_{ii} \le 1$  measures bilateral accessibility, G > 0 is a constant

• Third-country effects of trade costs, if any, are captured by the i and j terms.

### **Definitions**

Structural gravity model :

$$X_{ij} = \underbrace{\frac{Y_i}{\Omega_i}}_{S_i} \underbrace{\frac{X_j}{\Phi_j}}_{M_i} \phi_{ij} \tag{SG}$$

where

$$Y_i \equiv \sum_j X_{ij}$$
 (production)  
 $X_j = \sum_i X_{ij}$  (consumption)

$$X_j = \sum_i X_{ij}$$
 (consumption)

 $\Omega_i$  and  $\Phi_i$  are "multilateral resistance" terms :

$$\Phi_j = \sum_{l} \frac{\phi_{lj} Y_l}{\Omega_l}$$
 and  $\Omega_i = \sum_{l} \frac{\phi_{il} X_l}{\Phi_l}$ 

### **Definitions**

- Two assumptions in the structural model :
  - Spatial allocation of expenditures is independent of importer income :

$$\pi_{ij} \equiv rac{X_{ij}}{X_j} = rac{S_i \phi_{ij}}{\Phi_j}, \quad \textit{where} \quad \Phi_j = \sum_l S_l \phi_{lj}$$

Gravity estimates

 $\Phi_i$  captures consumers' choice set or competition in j

Goods market equilibrium :

$$Y_i = \sum_j X_{ij} = S_i \sum_j rac{X_j \phi_{ij}}{\Phi_j} \ \Rightarrow \ S_i = rac{Y_i}{\Omega_i}, \ \textit{where} \ \Omega_i = \sum_l rac{X_l \phi_{il}}{\Phi_l}$$

 $\Omega_i$  market potential in country i

See Appendix for details on the first assumption.

This is a selection of models that micro-found the gravity equation. See Head and Mayer (2014) p.18 for a more complete coverage.

### CES National Product Differentiation

- Anderson (1979)
- Iceberg trade costs
- CES utility :

$$U_{j} = \left[\sum_{i} \left(A_{i} q_{ij}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

- Armington assumption : each country *i* produces a different variety.
- Perfect competition.
- Gravity equation :

$$X_{ij} = \underbrace{\left(\frac{w_i}{A_i}\right)^{1-\sigma}}_{S_i} \underbrace{\frac{X_j}{P_j^{1-\sigma}}}_{M_i} \underbrace{\tau_{ij}^{1-\sigma}}_{\phi_{ij}}$$

### CES Monopolistic Competition

- Krugman(1980)
- Iceberg trade costs
- CES utility :

$$U_j = \left[\int \left(q_j(\omega)
ight)^{rac{\sigma-1}{\sigma}} d\omega
ight]^{rac{\sigma}{\sigma-1}}$$

- Monopolistic competition among  $N_i$  firms
- Gravity equation :

$$X_{ij} = \underbrace{\left(\frac{\sigma}{\sigma - 1}\right)^{1 - \sigma}}_{G} \underbrace{\left(N_{i} \frac{w_{i}}{\varphi_{i}}\right)^{1 - \sigma}}_{S_{i}} \underbrace{\frac{X_{j}}{P_{j}^{1 - \sigma}}}_{M.} \underbrace{\tau_{ij}^{1 - \sigma}}_{\phi_{ij}}$$

### Heterogeneous consumers, discrete choice

- Anderson et al. (1992)
- Assumptions
  - ▶  $L_j$  consumers with income  $w_j$ , indexed by I(j).
  - ▶ Heterogeneity in preferences over differentiated varieties :

$$u_{s(i)I(j)}(q_{s(i)I(j)}) = \ln[\psi_{s(i)I(j)}q_{s(i)I(j)}]$$

with  $\psi_{s(i)I(j)}$  an individual preference term distributed Fréchet and iid :

$$P[\psi_{s(i)I(j)} \le \psi] = e^{-\left(\frac{\psi}{A_i a_{ij}}\right)^{-\theta}}$$

 $\theta$  captures consumer heterogeneity,  $A_i$  and  $a_{ij}$  'location' parameters

- Iceberg trade costs
- Constant markups.
- ► Each country supplies *N<sub>i</sub>* varieties.



### Heterogeneous consumers, discrete choice

- Consumers choose **one** variety and spend  $\frac{w_j}{p_{s(i)j}}$ .
- Their choice maximizes

$$v_{s(i)l(j)}(q_{s(i)l(j)}) = \ln(w_j) - \ln(\mu w_i \tau_{ij}) + \ln(\psi_{s(i)l(j)})$$

where  $ln(\psi_{s(i)l(j)})$  is distributed Gumbel.

 $\Rightarrow$  The probability that good s(i) is bought by I(j) equals

$$Prob[v_{s(i)l(j)}(q_{s(i)l(j)}) \ge \max_{k} v_{s(k)l(j)}(q_{s(k)l(j)})] \equiv P_{ij} = \frac{w_{i}^{-\theta} A_{i}^{\theta} \tau_{ij}^{-\theta} a_{ij}^{\theta}}{\sum_{l} w_{l}^{-\theta} A_{l}^{\theta} \tau_{lj}^{-\theta} a_{lj}^{\theta}}$$

• Gravity equation :

$$X_{ij} = \underbrace{N_i w_i^{-\theta} A_i^{\theta}}_{S_i} \underbrace{\frac{w_j L_j}{\sum_{l} w_l^{-\theta} A_l^{\theta} \tau_{lj}^{-\theta} a_{lj}^{\theta}}}_{M_i} \underbrace{\tau_{ij}^{-\theta} a_{ij}^{\theta}}_{\phi_{ij}}$$

### Heterogeneous Industries

- Eaton & Kortum (2002)
- Assumptions
  - A continuum of "industries" heterogeneous in productivities

$$P[z_i \le z] = e^{-T_i z^{-\theta}}$$

- Perfect competition across countries
- Iceberg trade costs
- Gravity equation :

$$X_{ij} = \underbrace{T_i w_i^{-\theta}}_{S_i} \underbrace{\frac{X_j}{\sum_{l} T_l w_l^{-\theta} \tau_{lj}^{-\theta}}}_{M_i} \underbrace{\tau_{ij}^{-\theta}}_{\phi_{ij}}$$

- Melitz (2003) + Chaney (2008)
- Assumptions
  - ▶ A continuum of firms heterogeneous in productivities
  - Monopolistic competition across firms and countries
  - Iceberg trade costs
- Gravity equation :

$$X_{ij} = \underbrace{N_i w_i^{1-\sigma}}_{S_i} \underbrace{\frac{X_j}{\sum_{l} N_l w_l^{1-\sigma} \tau_{lj}^{1-\sigma} \tilde{\varphi}(\varphi_{lj}^*)^{\sigma-1}}_{M_i}} \underbrace{\tau_{ij}^{1-\sigma} \tilde{\varphi}(\varphi_{ij}^*)^{\sigma-1}}_{\phi_{ij}}$$

• With a Pareto distribution of productivities ( $G(\varphi) = 1 - \varphi^{-\theta}$ ) :

$$X_{ij} = \underbrace{N_i w_i^{1-\sigma}}_{S_i} \underbrace{\frac{X_j}{\sum_{l} N_l w_l^{1-\sigma} \tau_{lj}^{-\theta} f_{lj}^{-\left[\frac{\theta}{\sigma-1}-1\right]}}_{M_i} \underbrace{\tau_{ij}^{-\theta} f_{ij}^{-\left[\frac{\theta}{\sigma-1}-1\right]}}_{\phi_{ij}}$$

Theory-consistent estimation

- ullet In the Armington model,  $rac{d \ln X_{ij}}{d \ln au_{ii}} = -(\sigma-1)$ , a demand parameter
- In the context of heterogeneous consumers,  $\frac{d \ln X_{ij}}{d \ln au_{ii}} = -\theta$ , a demand parameter
- In the heterogeneous industries model,  $\frac{d \ln X_{ij}}{d \ln \tau_{ii}} = -\theta$ , a supply parameter
- In the heterogeneous firms model,  $\frac{d \ln X_{ij}}{d \ln \tau_{ii}} = -\theta$  and  $rac{d\ln X_{ij}}{d\ln f_{ii}} = -\left[rac{ heta}{\sigma-1}-1
  ight]$  , combination of demand and supply parameters

### Empirical challenges

- Historically, gravity equations were using as RHS variables the countries' GDP, populations and bilateral measures of barriers to trade
- This does not control for the "multilateral resistance terms" ( $\Phi_j$  and  $\Omega_i$ ) which creates a bias (Anderson & van Wincoop, 2003).
- Intuition: two small countries shut down from the ROW will spend half of their income on each other's imports, but much less if the same countries suddenly open to trade with 3rd countries.
- Various solutions have been proposed in the literature

#### Proxies for Multilateral Resistance Terms

 Log-GDP-weighted average distance (Wei, 1996, Baldwin & Harrigan, 2011):

$$Remoteness_{j} = \left(\sum_{i} \frac{Y_{i}}{Dist_{ij}}\right)$$

Larger for countries that are closer to large countries More or less consistent with the theory if  $\phi_{ii} = Dist_{ii}^{-1}$ ,  $X_i = Y_i$  and thus  $\Phi_j = \sum_k \frac{Y_l}{Dist_i} \Omega_l^{-1}$  and  $\Omega_i = \sum_l \frac{Y_l}{Dist_i} \Phi_l^{-1}$ 

- Iterative structural estimation (Head & Mayer, 2014) :
  - i) Assumes  $\Omega_i = 1$  and  $\Phi_i = 1$ , ii) Estimates the model to recover the parameters determining  $\phi_{ii}$ , iii) Given those parameters, compute new  $\Omega_i$ s and  $\Phi_i$ s, iv) Iterate until the parameters stop changing

Firm-level Gravity

#### Fixed effect estimations

Importer and Exporter fixed effects :

$$\ln X_{ij} = \ln G + \ln S_i + \ln M_j + \ln \phi_{ij}$$

Note: In panel data,  $S_i$  and  $M_i$  should also have a time-dimension. In sectoral data, they should also have the industry dimension.

Ratio-type estimations: use ratios to get rid of some fixed effects

$$\begin{split} \frac{X_{ij}}{X_{jj}} &= \frac{S_i}{S_j} \frac{\phi_{ij}}{\phi_{jj}}, \quad \frac{X_{ij}/X_{ik}}{X_{lj}/X_{lk}} = \frac{\phi_{ij}/\phi_{ik}}{\phi_{lj}/\phi_{lk}} \\ \frac{X_{ij}}{X_{jj}} \frac{X_{ji}}{X_{ii}} &= \frac{\phi_{ij}\phi_{ji}}{\phi_{jj}\phi_{ii}} \Rightarrow \phi_{ij} = \sqrt{\frac{X_{ij}X_{ji}}{X_{ii}X_{jj}}} \text{ if } \phi_{ij} = \phi_{ji} \text{ and } \phi_{ii} = 1 \\ \frac{X_{ij}X_{jk}X_{ki}}{X_{ji}X_{kj}X_{ik}} &= \left(\frac{(1+t_{ij})(1+t_{jk})(1+t_{ki})}{(1+t_{ji})(1+t_{kj})(1+t_{ik})}\right)^{\epsilon} \end{split}$$

where  $(1 + t_{ij})$  is the asymmetric component of trade costs



#### **7eros in Trade Matrices**

- Up to now, we have systematically considered gravity equations which are solely defined for strictly positive trade flows
- Helpman et al (2008): Even at the country level, about half the observations in the typical trade matrix are zeros
- The problem gets even worse in more disaggregated data
- How can models / estimation methods take this into account?
- Theoretical tricks: Truncate the productivity distribution (Helpman et al, 2008), Abandon the assumption of a continuum of firms (Eaton et al, 2012). Since zeros are more likely across distance/costly country pairs, neglecting those zeros will systematically underestimate the impact of distance

### Proposed solutions

- Use  $ln(1 + X_{ij})$  as LHS variable : A bad idea! Sensitive to units
- Eaton and Kortum (2001): Estimate a Tobit model where the LHS variable is defined as  $\ln X_{ij}^*$  where  $X_{ij}^* = X_{ij}$  for all positive trade flows and  $X_{ij}^* = \underline{X}_{ij}$  whenever  $X_{ij} = 0$ .  $\underline{X}_{ij}$  defined as the minimum value of trade for a given j. Amounts to assume that missing values are trade flows which fall below a declaration threshold
- Helpman et al (2008): Heckman-based approach: i) probit to estimate the probability of  $X_{ij} > 0$  and ii) OLS gravity equation on positive trade flows including a selection correction. Exclusion restriction: Overlap in religion and product of dummies for low entry barriers in countries i and j...
- Eaton et al (2012) : Multinomial PML deal with the zeros induced

Firm-level Gravity

## Meta-Analysis Results

$$\begin{array}{lll} \ln X_{ij} & = & \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln \textit{Dist}_{ij} + \alpha_4 \mathbf{1}_{\textit{Contiguity}_{ij}} + \alpha_5 \mathbf{1}_{\textit{CommonLanguage}_{ij}} \\ & & + \alpha_6 \mathbf{1}_{\textit{ColonialLink}_{ij}} + \alpha_7 \mathbf{1}_{\textit{RTA/FTA}_{ij}} + \alpha_8 \mathbf{1}_{\textit{EU}_{ij}} + \alpha_9 \mathbf{1}_{\textit{NAFTA}_{ij}} \\ & & + \alpha_9 \mathbf{1}_{\textit{CommonCurrency}_{ij}} + \alpha_{10} \mathbf{1}_{\textit{Home}_{ij}} + \varepsilon_{ij} \end{array}$$

All Gravity				Structural Gravity			
Median	Mean	s.d.	#	Median	Mean	s.d.	#
.97	.98	.42	700	.86	.74	.45	31
.85	.84	.28	671	.67	.58	.41	29
89	93	.4	1835	-1.14	-1.1	.41	328
.49	.53	.57	1066	.52	.66	.65	266
.49	.54	.44	680	.33	.39	.29	205
.91	.92	.61	147	.84	.75	.49	60
.47	.59	.5	257	.28	.36	.42	108
.23	.14	.56	329	.19	.16	.5	26
.39	.43	.67	94	.53	.76	.64	17
.87	.79	.48	104	.98	.86	.39	37
1.93	1.96	1.28	279	1.55	1.9	1.68	71
	.97 .85 89 .49 .49 .91 .47 .23 .39	Median         Mean           .97         .98           .85         .84          89        93           .49         .53           .49         .54           .91         .92           .47         .59           .23         .14           .39         .43           .87         .79	Median         Mean         s.d.           .97         .98         .42           .85         .84         .28          89        93         .4           .49         .53         .57           .49         .54         .44           .91         .92         .61           .47         .59         .5           .23         .14         .56           .39         .43         .67           .87         .79         .48	Median         Mean         s.d.         #           .97         .98         .42         700           .85         .84         .28         671          89        93         .4         1835           .49         .53         .57         1066           .49         .54         .44         680           .91         .92         .61         147           .47         .59         .5         257           .23         .14         .56         329           .39         .43         .67         94           .87         .79         .48         104	Median         Mean         s.d.         #         Median           .97         .98         .42         700         .86           .85         .84         .28         671         .67          89        93         .4         1835         -1.14           .49         .53         .57         1066         .52           .49         .54         .44         680         .33           .91         .92         .61         147         .84           .47         .59         .5         257         .28           .23         .14         .56         329         .19           .39         .43         .67         94         .53           .87         .79         .48         104         .98	Median         Mean         s.d.         #         Median         Mean           .97         .98         .42         700         .86         .74           .85         .84         .28         671         .67         .58          89        93         .4         1835         -1.14         -1.1           .49         .53         .57         1066         .52         .66           .49         .54         .44         680         .33         .39           .91         .92         .61         147         .84         .75           .47         .59         .5         257         .28         .36           .23         .14         .56         329         .19         .16           .39         .43         .67         94         .53         .76           .87         .79         .48         104         .98         .86	Median         Mean         s.d.         #         Median         Mean         s.d.           .97         .98         .42         700         .86         .74         .45           .85         .84         .28         671         .67         .58         .41          89        93         .4         1835         -1.14         -1.1         .41           .49         .53         .57         1066         .52         .66         .65           .49         .54         .44         680         .33         .39         .29           .91         .92         .61         147         .84         .75         .49           .47         .59         .5         257         .28         .36         .42           .23         .14         .56         329         .19         .16         .5           .39         .43         .67         .94         .53         .76         .64           .87         .79         .48         104         .98         .86         .39

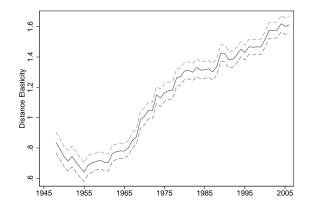
Notes: The number of estimates is 2508, obtained from 159 papers. Structural gravity refers here to some use of country fixed effects or ratio-type method.

### Meta-Analysis Results

- Average distance effect around -1.1
- Contiguity and common language effects around .5 (+65% of trade conditional on sharing a border or the same language). Colonial linkages imply larger effects (+130%)
- Some uncertainty regarding the impact of RTAs but NAFTA seems to have larger effects
- Estimates on common currency imply a doubling of trade, on average.
   Lower than the initial estimates by Rose (2000) who found a tripling of trade. Note that this does not control for the endogeneity of currency or trade unions
- Home bias is still huge, +370%



### Distance elasticity, over time



- Doubling the distance reduces trade by a factor of two
- Interpretation: Transportation costs, "Time as a trade barrier", Cultural distance, Informational frictions
- Over time, trade becomes more geographically concentrated!



Impact of changing trade barriers :

$$\frac{X'_{ij}}{X_{ij}} = \underbrace{\frac{\phi'_{ij}}{\phi_{ij}}}_{Direct} \underbrace{\frac{\Omega_i}{\Omega'_i} \frac{\Phi_j}{\Phi'_j}}_{QiRAdj. GDP Adj.} \underbrace{\frac{Y'_i}{Y_i} \frac{X'_j}{X_j}}_{QiRAdj. GDP Adj.}$$

- Direct impact :  $exp[\hat{\alpha}_i(B'_{ii} B_{ij})]$
- Impact on multilateral resistance indices: Usually negative. eg signing an RTA between i and j implies a decrease in  $\tau_{ii}$  (an increase in  $\phi_{ii}$ ). Because RTA makes access to *i* easier, competition gets fiercer and raises  $\Phi_i$ . This counteracts the direct effect of a raise in  $\phi_{ij}$  and transmit the impact of the shock on all the  $X_{i'i}$  terms
- Impact on GDPs is obtained through simulations.



### Partial, Modular and General Equilibrium Trade Impacts

Table 3.6 PTI, MTI, GETI, and Welfare Effects of Typical Gravity Variables

	Coeff.	PTI	MTI		GETI		Welfare	
Members:	Yes	Yes	Yes	No	Yes	No	Yes	No
RTA/FTA (all)	.28	1.323	1.129	.946	1.205	.96	1.011	.998
EU	.19	1.209	1.085	1.007	1.136	1.001	1.013	.999
NAFTA	.53	1.699	1.367	1.005	1.443	1	1.048	1
Common currency	.98	2.664	1.749	1.028	2.203	1.003	1.025	.998
Common language	.33	1.391	1.282	.974	1.303	.99	1.005	.999
Colonial link	.84	2.316	2.162	.961	2.251	.984	1.004	.999
Border effect	1.55	4.711	4.647	.938	3.102	.681	.795	n/a

Notes: The MTI, GETI, and welfare are the median values of the real/counterfactual trade ratio for countries relevant in the experiment.

- MTI usually smaller than PTI
- GETI close to MTI except for large shocks like removing the border
- Welfare impact is usually small (see Lecture 10)



Firm-level Gravity

Firm-level Gravity

# Firm-level gravity

#### Motivation

- Estimate the intensive and extensive margin responses of trade flows.
- Proposed decompositions :

$$\frac{\partial \ln X_{ij}}{\partial \ln \tau_{ij}} = \frac{\partial \ln N_{ij}}{\partial \ln \tau_{ij}} + \frac{\partial \ln \bar{x}_{ij}}{\partial \ln \tau_{ij}}$$

$$= \underbrace{\frac{\partial \ln N_{ij}}{\partial \ln \tau_{ij}}}_{Ext.Margin} + \underbrace{\frac{1}{\bar{x}_{ij}} \left( \int_{\phi_{ij}^{*}}^{+\infty} \frac{\partial \ln x_{ij}(\varphi)}{\partial \ln \tau_{ij}} x_{ij}(\varphi) \frac{g(\varphi)}{1 - G(\varphi_{ij}^{*})} d\varphi \right)}_{Int.Margin}$$

$$+ \underbrace{\frac{-\partial \ln G(\varphi_{ij}^{*})}{\partial \ln \varphi_{ij}^{*}} \frac{\partial \ln \varphi_{ij}^{*}}{\partial \ln \tau_{ij}} \left( \frac{x_{ij}(\varphi_{ij}^{*})}{\bar{x}_{ij}} - 1 \right)}_{Int.Margin}$$

Comp. Effect

Extensive margin: elasticity of number of exporters w.r.t. trade costs Intensive margin: elasticity of average shipments of incumbent firms Comp. Effect: entrants/exiters' TFP differs from incumbents'

### Melitz-Chaney model

• Intensive margin :

$$x_{ij}(\varphi) = \left(\frac{\sigma}{\sigma - 1} \frac{w_i \tau_{ij}}{\varphi}\right)^{1 - \sigma} \frac{X_j}{\Phi_j} \ \Rightarrow \ \frac{\partial \ln x_{ij}(\varphi)}{\partial \ln \tau_{ij}} = 1 - \sigma$$

Extensive margin :

$$N_{ij} = (1 - G(\varphi_{ij}^*))N_i \ \Rightarrow \ \frac{\partial \ln N_{ij}}{\partial \ln \tau_{ij}} = -\frac{\partial \ln G(\varphi_{ij}^*)}{\partial \ln \varphi_{ij}^*} \underbrace{\frac{\partial \ln \varphi_{ij}^*}{\partial \ln \tau_{ij}}}_{1}$$

Composition effect :

$$rac{-\partial \ln G(arphi_{ij}^*)}{\partial \ln arphi_{ii}^*} \left(rac{\mathsf{x}_{ij}(arphi_{ij}^*)}{ar{\mathsf{x}}_{ii}} - 1
ight)$$



### Melitz-Chaney model

Thus:

$$\frac{\partial \ln X_{ij}}{\partial \ln \tau_{ij}} = \underbrace{-\frac{\partial \ln G(\varphi_{ij}^*)}{\partial \ln G(\varphi_{ij}^*)}}_{Ext.Margin} + \underbrace{\frac{1 - \sigma}{\ln t.Margin}}_{Int.Margin} + \underbrace{\frac{-\partial \ln G(\varphi_{ij}^*)}{\partial \ln \varphi_{ij}^*} \left(\frac{x_i j(\varphi_{ij}^*)}{\bar{x}_{ij}} - 1\right)}_{Comp.Effect}$$

With Pareto :

$$\frac{\partial \ln X_{ij}}{\partial \ln \tau_{ij}} = \underbrace{-\theta}_{Ext.Margin} + \underbrace{1-\sigma}_{Int.Margin} + \underbrace{\sigma-1}_{Comp.Effect}$$

The composition and intensive margin effects offset each other exactly.

### Intensive and extensive gravity

Figure – The intensive & extensive components of the gravity equation (Crozet & Koenig, Table 2)

	All fir	ms	Single-region firms			
	> 20 emp	oloyees	> 20 employees			
	(1)	(2)	(3)	(4)		
	Average	Number of	Average	Number of		
	Shipment	Shipments	Shipment	Shipments		
	$\ln \left(M_{kjt}/N_{kjt}\right)$	$\ln (N_{kjt})$	$\ln \left(M_{kjt}/N_{kjt}\right)$	$\ln (N_{kjt})$		
$\ln (GDP_{kj})$	$0.461^{a}$	$0.417^{a}$	$0.421^{a}$	$0.417^{a}$		
	(0.007)	(0.007)	(0.007)	(0.008)		
$\ln \left( \mathrm{Dist}_{i} \right)$	$-0.325^a$	$-0.446^a$	$-0.363^a$	$-0.475^{a}$		
	(0.013)	(0.009)	(0.012)	(0.009)		
$Contig_j$	$-0.064^{c}$	-0.007	0.002	$0.190^{a}$		
	(0.035)	(0.032)	(0.038)	(0.036)		
Colony $_i$	$0.100^{a}$	$0.466^{a}$	$0.141^{a}$	$0.442^{a}$		
.,	(0.032)	(0.025)	(0.035)	(0.027)		
$French_i$	$0.213^{a}$	$0.991^{a}$	$0.188^{a}$	$1.015^{a}$		
,	(0.029)	(0.028)	(0.032)	(0.028)		
N	23553	23553	23553	23553		
$R^2$	0.480	0.591	0.396	0.569		

Note: These are OLS estimates with year and industry dummies. Robust standard errors in parentheses with ", b and c denoting significance at the 1%.5% and 10% level respectively.

 Extensive margin accounts for 57% of the distance effect. Larger share in other studies

#### Conclusion

- Nowadays, gravity is both a successful empirical model and a benchmark which guides theoretical modeling
- Krugman (1997): Gravity equations are examples of "social physics", the relatively few law-like empirical regularities that characterize social interactions.
- Gravity equation has also been used in other contexts, with some success:
  - Service offshoring (Head et al, 2009),
  - ► Migration (Anderson, 2011),
  - Commuting (Ahlfeldt et al, 2014),
  - Portfolio investments (Portes et al, 2001),
  - ► FDI (Head & Ries, 2008)



#### References

- Ahlfeldt, Redding, Sturm & Wolf, 2014, "The Economics of Density: Evidence from the Berlin Wall", NBER WP20354
- Anderson, 1979, "A theoretical foundation for the gravity equation", American Economic Review, 69(1):106-116
- Anderson, 2011, "The Gravity Model", The Annual Review of Economics, 3(1):133-160
- Anderson, de Palma, Thisse, 1992, Discret Choice Theory of Product Differentitation, MIT Press
- Anderson & van Wincoop, 2003, "Gravity with gravitas: A solution to the border puzzle", The American Economic Review 93(1):170-192
- Arkolakis, Costinot & Rodriguez-Clare, 2012. "New Trade Models, Same Old Gains?," American Economic Review, 102(1): 94-130
- Chaney, 2008, "Distorted Gravity: the intensive and extensive margins of international trade," American Economic Review, 98(4):1707-21

Micro-Foundations

- Eaton & Kortum, 2002, "Technology, geography and trade", Econometrica 70(5): 1741-1779
- Eaton, Kortum & Sotelo, 2012, "International Trade: Linking Micro and Macro", **NBER WP**
- Head, Mayer & Ries, 2009, "How remote is the offshoring threat?", European Economic Review, 53(4):429-444
- Head & Mayer, 2014, "Gravity Equations: Workhouse, Toolkit, and Cookbook", in Handbook of International Economics, Chapter 3
- Head & Ries, 2008, "FDI as an outcome of the market for corporate control: theory and evidence", Journal of International Economics 74(1): 2-20
- Helpman, Melitz & Rubinstein, 2008, "EStimating trade flows: trading partners and trading volumes", Quarterly Journal of Economics, 123(2):441-487
- Krugman, 1997, Development, Geography and Economic Theory, Vol 6 MIT Press
- McCallum, 1995, "National borders matter: Canada-US regional trade patterns," The American Economic Review 85(3): 615-623

#### References

- Melitz, 2003, "The impact of trade on intra-industry reallocations and aggregate industry productivity", Econometrica 71(6):1695-1725
- Melitz & Ottaviano, 2008, Market size, trade, and productivity", Review of Economic Studies 75(1):295-316
- Portes, Rey & Oh, 2001, Information and capital flows: the determinants of transactions in financial assets," European Economic Review 45(4-6):783-796
- Trefler, 1995, The case of missing trade and other mysteries," *The American Economic Review* 85(5): 1029-1046

Firm-level Gravity

• We showed that structural gravity (SG) requires two assumptions :

$$\pi_{ij} \equiv \frac{X_{ij}}{X_j} = \frac{S_i \phi_{ij}}{\Phi_j}, \quad \text{where} \quad \Phi_j = \sum_l S_l \phi_{lj}$$
 (1)

$$Y_i = \sum_j X_{ij} \tag{2}$$

which jointly imply

$$Y_i = S_i \sum_i \frac{X_j \phi_{ij}}{\Phi_j} \implies S_i = \frac{Y_i}{\Omega_i}, \text{ where } \Omega_i = \sum_l \frac{X_l \phi_{il}}{\Phi_l}$$

• We can show that *any* model that satisfies general gravity and trade balance in addition to market-clearing satisfies the first assumption.

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...assume

$$X_{ij} = S_i M_j \phi_{ij} \tag{GG}$$

$$Y_j - X_{jj} = \sum_{i \neq j} X_{ij} \Leftrightarrow Y_j = \sum_i X_{ij}$$
 (TB)

$$Y_i = \sum_i X_{ij} \tag{MC}$$

Combining (GG) and (TB) yields

$$X_{ij} = S_i \left( \frac{Y_j}{\sum_i S_i \phi_{ij}} \right) \phi_{ij} \Leftrightarrow \frac{X_{ij}}{\sum_i X_{ij}} = \frac{S_i \phi_{ij}}{\sum_i S_i \phi_{ij}}$$

which is exactly our budget share assumption (1).

• This equation can be combined with (MC) as before to yield :

$$X_{ij} = \frac{Y_i}{\Omega_i} \frac{X_j}{\Phi_i} \phi_{ij} \tag{SG}$$