International Trade core course
Master in Economics, 2nd year
Université Paris-Saclay

1 Heckscher-Ohlin-Vanek Empirics [20%]

- Several acceptable statements of the HOV theorem:
  - net exports are intensive in abundant factors
  - equivalently, the factor content of net exports is disproportionately high for abundant factors and disproportionately low for other factors
  - formally, with V factors and N products:

$$F^c = AT^c$$

    where F^c is the $(V, 1)$ vector of factor endowments in country c, T^c is the $(N, 1)$ vector of net exports, and A is the world technology matrix.

- Papers discussed in class include Leontief (1953) and Leamer (1980), Bowen et al. (1987), Trefler (1993), Trefler (1995), Davis and Weinstein (2001), Leamer (1984) and Harrigan (1995). See lecture 5 for details of each paper and in particular p16 for a summary table.

- Some missing features of HOV identified by its empirical failure [**non-exhaustive**]
  - empirical models with Hicks-neutral productivity differences (Trefler) or even factor intensity differences (Davis and Weinstein) outperform HOV, suggesting the need to study non-FPE (multi-cone) models.
  - empirical models with non-homothetic preferences and home bias in consumption also outperform HOV, suggesting the need to allow for other utility functions and trade costs
  - the low factor content of trade suggests the need for intra-industry models

2 Comparative Advantage and Firm Selection [80%]

1. In comparison with the Melitz framework, there are two important new assumptions:

   - Multiple sectors
   - Multiple factors of production

   The introduction of two sectors and two factors of production draws a line between the Melitz model and the HOS framework. Here, sectors differ in terms of their factoral intensities, with sector 1 being relatively intensive in capital while sector 2 is relatively intensive in labor. This, together with the fact countries differ by their relative factoral endowments, implies that there is a potential force towards sectoral specialization, with the home country being relatively rich in capital thus having a comparative advantage in the production of capital-intensive goods.

In comparison with Melitz, there are now two sources of heterogeneity across firms: Within a sector, firms differ by their productivity and across sectors, they differ by their factoral intensities. For instance, two equally productive firms belonging to different sectors might not end up with the same competitivity depending on the relative price of factors.

2. Autarky equilibrium.

   (a) Zero Profit Condition:
   Upon entry, firms draw their productivity in $g(\varphi)$. Conditional on their productivity, they have two options: i) exit immediately, which is optimal if producing would entail negative profits ($\pi(\varphi) < 0$), or ii) start producing and continue until being hit by a death shock (since productivity is constant over time) which is optimal if this entails positive profits ($\pi(\varphi) > 0$). Firms are indifferent between both strategies if:

$$
\begin{aligned}
& \pi(\varphi^*_{ic}) = 0 \\
\Leftrightarrow \quad & \frac{\alpha_i R_c}{\sigma}\left(\frac{p^d_{ic}(\varphi^*_{ic})}{P_{ic}}\right)^{1-\sigma} = f\rho_c^{\beta_i} w_c^{1-\beta_i} \\
\Leftrightarrow \quad & \frac{\alpha_i R_c}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{\rho_c^{\beta_i} w_c^{1-\beta_i}}{\varphi^*_{ic}}\frac{1}{P_{ic}}\right)^{1-\sigma} = f\rho_c^{\beta_i} w_c^{1-\beta_i} \\
\Leftrightarrow \quad & \varphi^*_{ic} = \frac{\sigma}{\sigma-1}\frac{\rho_c^{\beta_i} w_c^{1-\beta_i}}{P_{ic}}\left[\frac{\sigma}{\alpha_i R_c} f\rho_c^{\beta_i} w_c^{1-\beta_i}\right]^{\frac{1}{\sigma-1}}
\end{aligned}
$$

   (b) Free Entry Condition:
   Before knowing their probability, the expected value of entry is equal to the probability of successful entry times the expected profit, conditional on producing:

$$
(1-G(\varphi^*_{ic}))\sum_{t=0}^{\infty}(1-\delta)^t E(\pi^d_{ic}(\varphi)) = (1-G(\varphi^*_{ic}))\frac{\pi^d_{ic}(\tilde{\varphi_{ic}}(\varphi^*_{ic}))}{\delta}
$$

   where $\pi^d_{ic}(\tilde{\varphi_{ic}}(\varphi^*_{ic}))$ is the profit of a firm with the average productivity level $\tilde{\varphi_{ic}}(\varphi^*_{ic})$ The free entry condition states that, in equilibrium, the expected value of entry is equal to the sunk entry cost and thus:

$$
\begin{aligned}
& f^e\rho_c^{\beta_i} w_c^{1-\beta_i} = (1-G(\varphi^*_{ic}))\frac{\pi^d_{ic}(\tilde{\varphi_{ic}}(\varphi^*_{ic}))}{\delta} \\
\Leftrightarrow \quad & \frac{\delta f^e\rho_c^{\beta_i} w_c^{1-\beta_i}}{1-G(\varphi^*_{ic})} = \frac{\alpha_i R_c}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{\rho_c^{\beta_i} w_c^{1-\beta_i}}{\tilde{\varphi_{ic}}(\varphi^*_{ic})}\frac{1}{P_{ic}}\right)^{1-\sigma} - f\rho_c^{\beta_i} w_c^{1-\beta_i}
\end{aligned}
$$

(c) Combining both:

$$\begin{aligned}
\frac{\delta f^e \rho_c^{\beta_i} w_c^{1-\beta_i}}{1-G(\varphi_{ic}^*)} &= \left(\frac{\varphi_{ic}^*}{\tilde{\varphi_{ic}}(\varphi_{ic}^*)}\right)^{1-\sigma} \frac{\alpha_i R_c}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{\rho_c^{\beta_i} w_c^{1-\beta_i}}{\varphi_{ic}^*}\frac{1}{P_{ic}}\right)^{1-\sigma} - f\rho_c^{\beta_i} w_c^{1-\beta_i} \\
\Leftrightarrow \quad \frac{\delta f^e \rho_c^{\beta_i} w_c^{1-\beta_i}}{1-G(\varphi_{ic}^*)} &= \left[\left(\frac{\varphi_{ic}^*}{\tilde{\varphi_{ic}}(\varphi_{ic}^*)}\right)^{1-\sigma} - 1\right] f\rho_c^{\beta_i} w_c^{1-\beta_i} \\
\Leftrightarrow \quad \frac{\delta f^e}{f} &= (1-G(\varphi_{ic}^*))\left[\left(\frac{\tilde{\varphi_{ic}}(\varphi_{ic}^*)}{\varphi_{ic}^*}\right)^{\sigma-1} - 1\right] \\
\Leftrightarrow \quad \frac{\delta f^e}{f} &= \left[\int_{\varphi_{ic}^*}^{\infty}\left(\frac{\varphi}{\varphi_{ic}^*}\right)^{\sigma-1} g(\varphi)d\varphi - (1-G(\varphi_{ic}^*))\right] \\
\Leftrightarrow \quad \frac{\delta f^e}{f} &= \int_{\varphi_{ic}^*}^{\infty}\left[\left(\frac{\varphi}{\varphi_{ic}^*}\right)^{\sigma-1} - 1\right] g(\varphi)d\varphi
\end{aligned}$$

With a Pareto distribution:

$$\frac{\delta f_e}{f} = \left(\frac{\varphi_{min}}{\varphi_{ic}^*}\right)^k \left[\frac{k}{k-\sigma+1} - 1\right]$$

This equation uniquely determines φ_{ic}^* as a function of the fundamental parameters. Notice that the productivity cut-off is homogenous across countries and sectors. This is because the model assumes the same factoral intensity for both the sunk entry cost and the fixed cost of producing. As a consequence of this assumption, the factoral prices disappear from the free entry condition and the productivity cut-off only depends on the deep parameters of the model.

In equilibrium, the productivity cut-off depends on the relative size of the sunk entry cost and the fixed production cost, as well as the probability death, and the shape of the productivity distribution (k in the Pareto case). An increase in the fixed cost of producing means that firms must draw a higher productivity to make positive profits, thus an increase in φ^*. Instead, a higher probability death or a larger sunk entry cost reduces the mass of entrants into an industry, thus increasing the ex-post profitability and enabling less productive firms to enter.

3. (Free trade equilibrium)

(a) In the free trade equilibrium, domestic producers necessarily find it profitable to export to the foreign country since this entails no additional (fixed) costs while allowing the firm to produce at a larger scale (CES implies the foreign demand is necessarily strictly positive). As a consequence, the profits of the firm are now:

$$\begin{aligned}
\pi_{ic}(\varphi) &= \frac{r_{ic}^d(\varphi)}{\sigma} + \frac{r_{ic}^x(\varphi)}{\sigma} - f\rho_c^{\beta_i} w_c^{1-\beta_i} \\
&= \frac{\alpha_i R_c}{\sigma}\left(\frac{p_{ic}^d(\varphi)}{P_{ic}}\right)^{1-\sigma} + \frac{\alpha_i R_{c'}}{\sigma}\left(\frac{p_{ic}^x(\varphi)}{P_{ic'}}\right)^{1-\sigma} - f\rho_c^{\beta_i} w_c^{1-\beta_i}
\end{aligned}$$

where the price set on domestic and foreign sales is the same ($p^d_{ic}(\varphi) = p^x_{ic}(\varphi)$) because the marginal cost of serving both markets is equal. Since all firms export and set the same price in both markets, in equilibrium we have $P_{ic} = P_{ic'}$ and thus:

$$\pi_{ic}(\varphi) = \frac{\alpha_i R_c}{\sigma} \left(\frac{p^d_{ic}(\varphi)}{P_{ic}}\right)^{1-\sigma} \left(1 + \frac{R_{c'}}{R_c}\right) - f \rho_c^{\beta_i} w_c^{1-\beta_i}$$

The increase in the firm's scale only depends on the relative size of the foreign country ($R_{c'}/R_c$).

(b) While all firms in both sectors export, countries still specialize along their comparative advantages. In autarky, the relative abundance of the home country in factor K implies that the relative price of capital is low in equilibrium, in comparison with the foreign country. In autarky, the home country thus enjoys a relatively low price for good 1, which production is relatively more intensive in capital:

$$\frac{P^a_{1H}}{P^a_{2H}} < \frac{P^a_{1F}}{P^a_{2F}}$$

When both countries start trading together, the home country specializes in good 1, i.e. exporters in sector 1 tend to sell relatively more abroad than exporters in sector 2. Such specialization patterns induce a convergence of relative prices across countries, and a convergence of factor prices. Factor price equalization is total if relative endowments are sufficiently close together.

4. (Costly trade)

(a) In the costly trade equilibrium, the two productivity cut-offs are obtained using the two zero cut-off profit conditions:

$$\frac{r^d_{ic}(\varphi^*_{ic})}{\sigma} = \frac{\alpha_i R_c}{\sigma} \left(\frac{p^d_{ic}(\varphi^*_{ic})}{P_{ic}}\right)^{1-\sigma} = f \rho_c^{\beta_i} w_c^{1-\beta_i}$$
$$\frac{r^x_{ic}(\varphi^{x*}_{ic})}{\sigma} = \frac{\alpha_i R_{c'}}{\sigma} \left(\frac{p^x_{ic}(\varphi^{x*}_{ic})}{P_{ic'}}\right)^{1-\sigma} = f^x \rho_c^{\beta_i} w_c^{1-\beta_i}$$

Using the fact that $p^x_{ic}(\varphi) = \tau p^d_{ic}(\varphi)$ and $r^x_{ic}(\varphi) = r^d_{ic}(\varphi)\tau^{1-\sigma}\frac{R_{c'}}{R_c}\left(\frac{P_{ic}}{P_{ic'}}\right)^{1-\sigma}$, one can rewrite the second ZCP condition as follows:

$$\frac{r^x_{ic}(\varphi^{x*}_{ic})}{\sigma} = f^x \rho_c^{\beta_i} w_c^{1-\beta_i}$$
$$\Leftrightarrow \quad \frac{r^d_{ic}(\varphi^{x*}_{ic})}{\sigma}\tau^{1-\sigma}\frac{R_{c'}}{R_c}\left(\frac{P_{ic}}{P_{ic'}}\right)^{1-\sigma} = f^x \rho_c^{\beta_i} w_c^{1-\beta_i}$$

Combining both together:

$$
\begin{aligned}
& \frac{r_{ic}^d(\varphi_{ic}^{x*})}{r_{ic}^d(\varphi_{ic}^*)}\tau^{1-\sigma}\frac{R_{c'}}{R_c}\left(\frac{P_{ic}}{P_{ic'}}\right)^{1-\sigma} = \frac{f^x}{f} \\
\Leftrightarrow\ & \left(\frac{\varphi_{ic}^{x*}}{\varphi_{ic}^*}\right)^{\sigma-1}\tau^{1-\sigma}\frac{R_{c'}}{R_c}\left(\frac{P_{ic}}{P_{ic'}}\right)^{1-\sigma} = \frac{f^x}{f} \\
\Leftrightarrow\ & \varphi_{ic}^{x*} = \varphi_{ic}^*\tau\frac{P_{ic}}{P_{ic'}}\left(\frac{R_c}{R_{c'}}\frac{f^x}{f}\right)^{\frac{1}{\sigma-1}}
\end{aligned}
$$

Since $(\tau)^{\sigma-1}f^x > f$, everything else equal, we have $\varphi_{ic}^{x*} > \varphi_{ic}^*$ and thus only the most productive firms export in equilibrium, while medium productive firms sell all their production to the domestic market. Note however that, with asymmetric countries, it might not be the case since a sufficiently small revenue in country c relative to country c' could imply $\varphi_{ic}^{x*} < \varphi_{ic}^*$. Since selection in export is consistent with empirical evidence, one can assume that $\varphi_{ic}^{x*} > \varphi_{ic}^*$ outside the symmetric equilibrium as well.
From this, one can show that the export threshold tends to be high relative to the zero-profit productivity cut-off (i.e. the export probability tend to be low) when: i) the fixed cost of exporting is high relative to the fixed cost of producing (f^x/f high), ii) the domestic price index is high relative to the foreign price index ($P_{ic}/P_{ic'}$ high), iii) the domestic revenue is high relative to the foreign one ($R_c/R_{c'}$ high). When the fixed cost of exporting is high and/or the export destination is small and highly competitive, exporting firms need to make sufficiently large profits in order to cover the fixed export cost, which requires being sufficiently productive.

(b) In the costly trade equilibrium:

$$
\frac{f}{\delta}\left[\int_{\varphi_{ic}^*}^{+\infty}\left[\left(\frac{\varphi}{\varphi_{ic}^*}\right)^{\sigma-1}-1\right]dG(\varphi)\right]+\frac{f_x}{\delta}\left[\int_{\varphi_{ic}^{x*}}^{+\infty}\left[\left(\frac{\varphi}{\varphi_{ic}^{x*}}\right)^{\sigma-1}-1\right]dG(\varphi)\right] = f_e
$$

while under autarky:

$$
\frac{f}{\delta}\left[\int_{\varphi_{ic}^*}^{+\infty}\left[\left(\frac{\varphi}{\varphi_{ic}^*}\right)^{\sigma-1}-1\right]dG(\varphi)\right] = f_e
$$

The expected value of entry under costly trade equals the expected value of entry under autarky plus a second term reflecting the expected profits to be derived from serving the export market. Since the integral is decreasing in the domestic cutoff, the productivity threshold for making positive profits in the domestic market must be higher in the costly trade equilibrium than in autarky, which is indeed what Figure 1 displays.

(c) The change in the productivity cut-off is found to be larger in the comparative advantage industry, which also displays a lower export productivity cut-off. This also implies that opening to international trade has a larger impact on the mean productivity of firms in the comparative advantage industry. Also, the export productivity cut-off is closer to the zero-profit productivity cut-off in the comparative advantage industry. This is because exporting is relatively more attractive in comparative advantage industries, since firms have high expected profits there, conditional on exporting (because of their cost advantage over domestic firms). As a consequence, one should observe that firms have a higher export propensity in sector 1 of country H (sector 2 of country F). **Finally, using question 4a, one notes that:**

$$\frac{\varphi_{ic}^{x*}/\varphi_{ic}^{*}}{\varphi_{i'c}^{x*}/\varphi_{i'c}^{*}} = \frac{P_{ic}/P_{ic'}}{P_{i'c}/P_{i'c'}}$$

i.e. the relative export propensity of firms across sectors depends on the equilibrium relative sectoral prices and comparative advantage. Overall, factor trade liberalization causes factor reallocation and even more so in comparative advantage industries.

5. (Empirical Evidence)

   (a) Table 1 compares the labor intensity and export probabilities in two French industries, namely textile and pharmaceutical. The texile industry is relatively more labor intensive while the pharmaceutical industry uses more capital (and thus represents sector 1 in the model above). French firms are more likely to export in the pharmaceutical industry, which might be consistent with France having a comparative advantage in this sector:

$$\frac{1-G(\varphi_{1FRA}^{x*})}{1-G(\varphi_{1FRA}^{*})} > \frac{1-G(\varphi_{2FRA}^{x*})}{1-G(\varphi_{2FRA}^{*})}$$

   In order to confront the model with the data, it is however convenient to focus on the bilateral dimension. France is first compared with countries which are relatively labor abundant then with capital-abundant countries. Export probabilities show that the larger propensity of firms in the pharmaceutical industry to export is the strongest for interactions with labor intensive countries while export propensities are more comparable across sectors, when focusing on trade with capital-abundant countries.

   This is consistent with the predictions of the model. In the interactions between France and labor-abundant countries, France clearly has a comparative advantage in the production of capital-intensive goods. Such comparative advantage makes France specialize which, in the data, shows up in the stronger propensity of firms in the capital-intensive sector to export.

   (b) Results in Table 2 however suggests that a large source of heterogeneity in labor intensities is not across sectors, as supposed in the model, but across firms, within a sector. In particular, firms which export are significantly less labor-intensive than their domestic counterparts. This is all the more true in the comparative disadvantaged sector. This might suggest that firms manage to exporting by exploiting France's comparative advantage in capital. In order to take this into account, the model would need to be extended to endogenous choices of labor intensities.