# VII - The Standard Models of Trade Theory Under Imperfect Competition

- Purpose of this chapter:
  - → to present the standard trade models under imperfect competition
  - → to illustrate the various effects of free trade under imperfect competition presented in chapter 7
- Two different approaches presented here:
  - $\rightarrow$  1 : monopolistic competition à la Dixit-Stiglitz (Krugman [1980])
  - → 2: Cournot competition on homogenous products and segmented markets (Brander [1981])

# Part 2. Cournot Competition with Homogenous Goods

- Introduced by Brander [1981]
- Role of strategic interactions (a player strategy depends on the other player ones)
- Allow to consider the pro-competitive and efficiency effects of trade (mark-up are not constant) under imperfect competition, contrary to the DSK model
- 2 country model that can be generalized to N countries and to non-homogenous products.

# 1.1 The Equilibrium Under Autarky

- 1 homogenous good, no product differentiation
- Produced by  $n_i$  identical firms in country i
- Firm technology (same assumptions as in DSK)
  - $\rightarrow$  constant labor productivity:  $g_i$ 
    - $\Rightarrow$  production:  $q_i = g_i l_i$ , where  $l_i$  is labor per firm
  - $\rightarrow$  fixed cost:  $f_i$
- Factor and good price: labor cost:  $w_i$ ; good price:  $p_i$
- Firm profit:  $\pi_i = p_i q_i w_i l_i f_i = \left(p_i \frac{w_i}{g_i}\right) q_i f_i$

**Marginal cost** 

• Consumer preferences: quasi linear / quadratic by (simplifying) assumption (many other possibilities)

$$U_i = z_i + a_i * q_i^d - \frac{(q_i^d)^2}{2}$$
.

 $z_i$  homogeneous good produced under perfect competition

Budget constraint is  $income = z_i + p_i * q_i^d$ 

Max  $U_i$  w.r.t  $z_i$  under budget constraint; equivalent to max:

$$income - p_i * q_i^d + a_i * q_i^d - \frac{(q_i^d)^2}{2}$$
 w.r.t.  $q_i^d$ .

- $\Rightarrow$  linear demand:  $q_i^d = a_i p_i$ 
  - ✓  $a_i$ : positive constant  $\Leftrightarrow$  Demand size

✓ note: non constant demand elasticity that decreases with the quantity:

$$\varepsilon = -\frac{dq}{dp} = -\frac{dq}{dp} \frac{p}{q} = \frac{p}{a-p} = \frac{a-q}{q} = \frac{a}{q} - 1$$

- Cournot competition: imperfect competition in quantity (Nash equilibrium) on homogenous goods.
- Assumption: simultaneous choice.
- First, short-run equilibrium characterization: the number of firms is exogenously fixed (such that profits are non-negative).
- Each firm maximizes its profit w.r.t. the quantity produced, taking into account the demand elasticity, assuming that the other firms hold their quantity constant.
- Market equilibrium:  $q_i^d = n^i q_i \implies p_i = a_i n_i q_i$

- But a given firm maximizes its profits w.r.t  $\tilde{q}_i$  such that  $p_i = a_i \tilde{q}_i (n_i 1)q_i$ 
  - $\Rightarrow \text{ firm program: } \underbrace{Max}_{\widetilde{q}_i} \left[ \left( a_i \frac{w_i}{g_i} (n_i 1)q_i \widetilde{q}_i \right) \widetilde{q}_i f \right]$
- First order condition:  $a_i \frac{w_i}{g_i} (n_i 1)q_i \tilde{q}_i \tilde{q}_i = 0$
- Best-response (optimal quantity for given quantities produced by

competitors): 
$$\tilde{q}_i = \frac{1}{2} \left( a_i - \frac{w_i}{g_i} - (n_i - 1)q_i \right)$$

• Nash equilibrium (intersection of the best-responses):

→ all firms produce the same quantity since identical

$$\Rightarrow \text{ quantity: } q_i = \frac{a_i - \frac{w_i}{g_i}}{1 + n_i}$$

$$\Rightarrow \text{ equilibrium price: } p_i = \frac{a_i + n_i \frac{w_i}{g_i}}{1 + n_i}$$

#### Note:

→ firms produce in equilibrium, if demand is large enough compared to marginal cost ⇔

$$q_i > 0 \Leftrightarrow a_i > \frac{w_i}{g_i}$$
 (assumed)

- $\Rightarrow$  non negative marginal profit:  $p_i \ge \frac{w_i}{g_i}$
- **⇒** positive mark-up

$$\rightarrow \text{mark-up:} \frac{p_i - \frac{w_i}{g_i}}{p_i} = \frac{a_i - \frac{w_i}{g_i}}{a_i + n_i \frac{w_i}{g_i}}$$

- ⇒ variable mark-up (contrary to DSK)
- ⇒ the mark-up decreases with the firm number

- → strategic interactions: the best-response is not a constant function but depends on the other firm strategies (their produced quantity here)
  - ✓ in DSK, the best-response is constant, independent of the variety number: no strategic interaction.

## Profits, consumer surplus and welfare

$$\rightarrow$$
 aggregate firm profits:  $\Pi_i = n_i \pi_i = \frac{n_i \left(a_i - \frac{w_i}{g_i}\right)^2}{(1 + n_i)^2} - n_i f_i$ 

# → consumer surplus:

## For a given firm:

$$S_i = \int_0^{q_i} (a_i - u) du - p_i q_i = \int_{p_i}^{a_i} (a_i - u) du = \frac{1}{2} (a_i - p_i) q_i = \frac{1}{2} (a_i - p_i)^2$$

#### For $n_i$ firmes :

$$S_{i} = \frac{n_{i}^{2} \left(a_{i} - \frac{w_{i}}{g_{i}}\right)^{2}}{2(1 + n_{i})^{2}}$$

#### $\rightarrow$ total welfare:

$$W_{i} = S_{i} + \Pi_{i} = \frac{n_{i} (2 + n_{i}) \left( a_{i} - \frac{w_{i}}{g_{i}} \right)^{2}}{2(1 + n_{i})^{2}} - n_{i} f_{i}$$

## • Country comparisons:

the larger [higher  $a_i$  (demand),  $n_i$  (firm number)],

the more efficient [higher  $g_i$  (productivity), lower  $f_i$  (fixed

costs), lower  $w_i$  (wage)] the country,

the higher the welfare under autarky.

- Long run equilibrium: endogenous firm number
  - $\rightarrow$  the zero profit condition gives the firm number:

$$\pi_i = 0 \Rightarrow n_i = \frac{a_i - \frac{w_i}{g_i}}{\sqrt{f_i}} - 1$$

- **⇔** maximum firm number such that non-negative profits
- ✓ it is higher, the lower the economies of scale (low fixed cost)

→ long-run welfare:

$$W_i = S_i = \frac{\left(a_i - \frac{w_i}{g_i} - \sqrt{f_i}\right)^2}{2}$$

**✓** larger than the short-run welfare

# 2.2 The Equilibrium Under Open Economy

- 2 countries, 1 and 2, producing exactly the same good (polar assumption compared to DSK)
- In each country, same preferences and technology as in the autarky case (here assume  $g_i = g_i = 1$ ).

• In particular, total number of firms producing the good:  $n_1 + n_2$  (short run assumption to begin with)

- Segmented market assumption:
  - → firms sell the good both on local and foreign markets, but incur a trade cost when exporting (payed by the firm, contrary to DSK)

 $\checkmark$  the trade cost t is assumed to be additive.

 $\rightarrow$  consumers buy the good only on their local market.

- → firms separately choose the quantity they sell on each market:
  - $\checkmark q_{ii}$  is the quantity produced by a firm located in country i and sold locally
  - $\checkmark q_{ij}$  is the quantity produced by a firm located in country i and exported to j.
- Firm profit:

$$\pi_{i} = p_{i}q_{ii} + p_{j}q_{ij} - tq_{ij} - w_{i}l_{i} - f_{i}$$
$$= (p_{i} - w_{i})q_{ii} + (p_{j} - w_{i} - t)q_{ij} - f_{i}$$

• First, short-run equilibrium characterization: the number of firms is exogenously fixed (such that profits are non-negative)

• Same Cournot-Nash equilibrium concept as under autarky:

$$\Rightarrow$$
 firm program:  $\underset{\widetilde{q}_{ii},\widetilde{q}_{ij}}{\mathit{Max}} [\pi_i (\widetilde{q}_{ii},\widetilde{q}_{ij},q_{ii},q_{ij},q_{ji},q_{ji})]$  with

$$\pi_{i}(\widetilde{q}_{ii}, \widetilde{q}_{ij}, q_{ii}, q_{ji}, q_{ji}) =$$

$$(a_{i} - w_{i} - (n_{i} - 1)q_{ii} - n_{j}q_{ji} - \widetilde{q}_{ii})\widetilde{q}_{ii} +$$

$$(a_{j} - w_{i} - t - (n_{i} - 1)q_{ij} - n_{j}q_{jj} - \widetilde{q}_{ij})\widetilde{q}_{ij} - f_{i}$$

• Best-response (optimal quantity for given quantities produced by competitors):

$$\begin{cases} \widetilde{q}_{ii} = \frac{1}{2} (a_i - w_i - (n_i - 1)q_{ii} - n_j q_{ji}) \\ \widetilde{q}_{ij} = \frac{1}{2} (a_j - w_i - t - (n_i - 1)q_{ij} - n_j q_{jj}) \end{cases}$$

• Nash equilibrium (intersection of the best-responses) when interior for both countries:

$$\Rightarrow \textbf{quantities:} \begin{cases} q_{ii} = \frac{a_i - w_i - n_j (w_i - (w_j + t))}{1 + n_1 + n_2} \\ q_{ij} = \frac{a_j - w_i - t - n_j (w_i + t - w_j)}{1 + n_1 + n_2} \end{cases}$$

$$\rightarrow$$
 equilibrium price:  $p_i = \frac{a_i + n_i w_i + n_j (w_j + t)}{1 + n_1 + n_2}$ 

## • Possible corner equilibrium:

$$\rightarrow q_{ii} > 0 \Leftrightarrow a_i - w_i - n_j (w_i - (w_j + t)) > 0$$

✓ if 
$$w_i < (w_i + t)$$
, always true

$$\checkmark$$
 if  $w_i > (w_j + t) \Leftrightarrow n_j < \frac{a_i - w_i}{w_i - (w_j + t)}$ 

$$\rightarrow q_{ij} > 0 \Leftrightarrow a_i - w_i - t - n_j (w_i + t - w_j) > 0$$

✓ if 
$$w_i + t < w_j$$
, always true

$$\checkmark$$
 if  $w_i + t > w_j$ ,  $\Leftrightarrow n_j < \frac{a_i - w_i - t}{w_i + t - w_j}$ 

# • Full equilibrium characterization:

 $\rightarrow$  case 1: low cost / productivity differences,

$$\Leftrightarrow w_j - t < w_i < w_j + t \Rightarrow$$

- ✓ firms always produce for their local market:  $q_{ii} > 0$
- ✓ firms export if the number of firms located in the other region is

**not too large:** 
$$q_{ij} > 0 \Leftrightarrow n_j < \frac{a_i - w_i - t}{w_i + t - w_j}$$

- $\rightarrow$  case 2: country j has a strong productive advantage  $\Leftrightarrow w_j + t \le w_i$ 
  - $\checkmark$  firms in country j always produce for both markets:

$$q_{jj} > 0$$
 and  $q_{ji} > 0$ 

 $\checkmark$  firms in country i export if the number of firms located in the

other region is not too large: 
$$q_{ij} > 0 \Leftrightarrow n_j < \frac{a_i - w_i - t}{w_i + t - w_j}$$
 and may

even not produce at all if this number is very large:

$$q_{ii} > 0 \Leftrightarrow n_j < \frac{a_i - w_i}{w_i - (w_j + t)}$$

✓ note: clearly, 
$$\frac{a_i - w_i - t}{w_i + t - w_j} < \frac{a_i - w_i}{w_i - (w_j + t)}$$

- <u>Long-run</u> equilibrium: # of firms producing in each country is endogenously determined such that profits are zero:  $\pi_1 = \pi_2 = 0$ 
  - ✓ multiple equilibria may exist, in particular when markets are perfectly integrated and countries are identical.

- Important conclusion: as soon as markets are segmented, t > 0, and the firm number is not too large, cross trade exists, even if the good is homogenous, and even if countries are identical
  - $\Rightarrow$  real intra-trade only due to imperfect competition.
  - ✓ note: in the long-run equilibrium under segmented markets, no intra-trade.

# 2.3 Trade Gains and Losses

- Assumption of an interior equilibrium (low market segmentation, low asymmetries between countries)
- <u>Short-run</u> equilibrium: fixed and exogenous firm number in both countries

## • Consumer surplus:

→ surplus variation: same sign as the price variation

$$\Rightarrow S_i^* - S_i^a = \left(a_i + \frac{p_i^* + p_i^a}{2}\right) \left(p_i^a - p_i^*\right)$$

 $\rightarrow$  price variation

$$\frac{p_i^*}{p_i^a} = \frac{1 + n_i}{1 + n_1 + n_2} \cdot \left(1 + \frac{n_j(w_j + t)}{a_i + n_i w_i}\right)$$

- ✓ decrease thanks to the pro-competitive effect,  $\frac{1+n_i}{1+n_1+n_2}$
- ✓ the larger  $n_i$ , the larger the decrease.

## • Firm profit variation

 $\rightarrow$  notation:  $\Pi_i = \Pi_{ii} + \Pi_{ij} - n_i f_i$ , where  $\Pi_{ij}$  is the variable profit on market j of firms located in i

$$\frac{n_{i}(a_{i}-w_{i}+n_{j}(w_{j}+t-w_{i}))^{2}}{\Pi_{ii}^{a}} = \frac{(1+n_{1}+n_{2})^{2}}{n_{i}(a_{i}-w_{i})^{2}} \\
\frac{\Pi_{ii}^{*}}{\Pi_{ii}^{a}} = \left(\frac{1+n_{i}}{1+n_{1}+n_{2}}\right)^{2} \left(1+\frac{n_{j}(w_{j}+t-w_{i})}{a_{i}-w_{i}}\right)^{2}$$

→ the double effect of trade on local profit

✓ pro competitive effect: 
$$\frac{1+n_i}{1+n_1+n_2}$$

the firm number increases, the mark-up decreases, the profit reduces.

**✓** production and rent shifting effect

$$1 + \frac{n_j(w_j + t - w_i)}{a_i - w_i}$$

- ✓ if  $w_j + t < w_i$ , new competitors are more productive than local producers, which decreases the local profit
- ✓ if  $w_j + t > w_i$ , new competitors are less productive than local producers, which increases the good price and next the local profit (dominated effect in general)
- ✓ lower profit loss (compared to autarky), the higher the trade cost (closes the market)

→ new export profit (if markets not too segmented)

$$\Pi_{ij}^* = \frac{n_i \left( a_j - (w_i + t) + n_j \left( w_j - (w_i + t) \right) \right)^2}{(1 + n_1 + n_2)^2}$$

✓ larger, the larger the export country, the higher the local productivity, the lower the competitors productivity, the lower the trade cost, the lower the competitor number (except if strong competitive advantage)

## • Special cases

 $\rightarrow$  same productivity  $(w_1 = w_2 = w)$ 

$$\frac{\prod_{ii}^{*}}{\prod_{ii}^{a}} = \left(\frac{1+n_i}{1+n_1+n_2}\right)^2 \left(1+\frac{n_j t}{a_i-w}\right)^2$$

pro-competitive effect stronger (⇒ local profit loss), unless market segmentation is large

may be compensated by the export profit (if export country large etc...)

 $\rightarrow$  if, moreover, same country size (demand  $(a_1 = a_2)$  and firm number  $(n_1 = n_2)$ ) and perfectly integrated markets (t = 0):

$$\checkmark \frac{\prod_{i}^{*} - n_{i} f_{i}}{\prod_{i}^{a} - n_{i} f_{i}} = 2 \left(\frac{1+n}{1+2n}\right)^{2} < 1: loss$$

$$\checkmark \frac{sc_i^*}{sc_i^a} = 4\left(\frac{1+n}{1+2n}\right)^2 > 1$$
: gain

$$\checkmark \frac{W_i^* - n_i f_i}{W_i^a - n_i f_i} = \frac{4(1+n)^3}{(2+n)(1+2n)^2} > 1: \text{ gain}$$

⇒ only the pro-competitive effect (direct deadweight loss decrease + efficiency gains)

#### Conclusions

- → to determinate which effects dominate when trade is liberalized under imperfect competition, one has to work on the analytics.
- → do not forget: partial equilibrium analysis
  - ✓ e.g., if profits are zero, the country does not care on having or not firms located in
  - ✓ under general equilibrium: this becomes critical (no firms ⇒ no employment ⇒ no income)
- → liberalization effects under imperfect competition and general equilibrium: see economic geography

#### • References

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