VII - Standard Models of Trade Theory Under Imperfect Competition

Part 1: The Krugman Model

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Introduction

- The Krugman (1980) model illustrates gains from trade that result from increased product variety.
- This complements our analysis of economies of scale and the pro-competitive effect of trade.
- Extension of the closed-economy model of Dixit and Stiglitz (1977).
- The model predicts how prices, quantities, number of varieties, wages and welfare are affected by trade liberalization.

Consumption (Dixit-Stiglitz)

- Representative household supplying L units of labor and owning all firms.
- CES preferences over a continuum of **varieties** Ω:

$$\max_{q(\omega)} U = \max_{q(\omega)} \left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$
$$\text{s.t.} \quad \int_{\Omega} p(\omega) q(\omega) d\omega = wL$$

  with $\sigma > 1$ the **elasticity of substitution** between varieties.
- Utility maximization yields the **demand function**:

$$q(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{wL}{P}$$

  with $P = \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$

▸ Details on the demand function

Interpretation of P: ideal price index

- P is the "ideal price index", in the sense that an extra unit of utility costs P extra units of income.
- Proof: plug demand functions into the utility function

$$
\begin{aligned}
U &= \left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} = \left(\int_{\Omega} \left(\frac{p(\omega)}{P}\right)^{-\sigma\frac{\sigma-1}{\sigma}} \left(\frac{wL}{P}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} \\
&= \frac{wL}{P} P^{\sigma} \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega\right)^{\frac{\sigma}{\sigma-1}} = \frac{wL}{P}
\end{aligned}
$$

- If both nominal income wL and the price index P increase by x%, utility remains unchanged.

Interpretation of P: love of variety

- No matter how high the price of variety ω, there will be some positive demand for ω.
- Since $\sigma > 1$, the ideal price index P is lower than the simple average of prices $p(\omega)$:
  $P = \left(\int_\Omega p(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}} < \int_\Omega p(\omega) d\omega$
- This captures the consumer's love of variety: consuming all varieties in the optimal bundle gives more utility than consuming a single variety at the average price.
- For a given nominal income wL and average price, increased product diversity lowers price index P and increases welfare.

- Each firm has **monopoly** over a variety ω which is imperfectly substitutable with other varieties (monopolistic competition).
- **Fixed cost**: to produce $q(\omega)$ firms need $f + \frac{q(\omega)}{\varphi}$ labor units
- **Optimal price**: $p = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}$ ▸ Details on the optimal price
- **Profit**: $\pi(\omega) \equiv p(\omega)q(\omega) - w\left(f + \frac{q(\omega)}{\varphi}\right) = w\left(\frac{q(\omega)}{(\sigma-1)\varphi} - f\right)$
- **Free entry**: $\pi(\omega) = 0 \Rightarrow q(\omega) = (\sigma-1)\varphi f$

$\Rightarrow$ All firms have the same quantity and price (ω now omitted)

- **Labor market equilibrium**: n such that

$$n\left(f + \frac{q}{\varphi}\right) = L \Rightarrow n = \frac{L}{\sigma f}$$

$\Rightarrow$ The number of firms increases with market size (L) and decreases with fixed costs (f) and competition (σ).

Back to the price index

- Equilibrium price index:

$$P = \left(\int_{\Omega} \left(\frac{\sigma}{\sigma - 1} \frac{w}{\varphi} \right)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} n^{\frac{1}{1-\sigma}}$$

  is decreasing in the number of varieties

- At the equilibrium value of n:

$$P = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} \left(\frac{L}{\sigma f} \right)^{\frac{1}{1-\sigma}}$$

- Larger economies have lower P's and higher welfare in autarky.

Opening the economy

- Consider two identical countries except for their size: L, L^*.
- Transport costs are of the Samuelson "iceberg" type: when 1 unit is shipped, $1/\tau$ units is received, with $\tau \geq 1$. $\tau - 1$ represents the ad valorem trade cost.
- Optimal prices
  - Domestic market: $p^D = \frac{\sigma}{\sigma-1}\frac{w}{\varphi} \equiv p$
  - Foreign market: $p^X = \tau\frac{\sigma}{\sigma-1}\frac{w}{\varphi} = \tau p$
- The price before transport (FOB) is the *same* on both markets. The price at destination (CIF) includes the transport cost τ, which is fully passed on to the consumer.

Equilibrium in the Open Economy

- Prices $p^D = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}$ and $p^X = \tau\frac{\sigma}{\sigma-1}\frac{w}{\varphi}$
- Total production: $q = q^D + \tau q^X$
- Total profit: $\pi = (p - \frac{w}{\varphi})q^D + (\tau p - \tau\frac{w}{\varphi})q^X - wf = pq - w\left(f + \frac{q}{\varphi}\right) = \frac{w}{(\sigma-1)\varphi}q - wf$
- Free entry: $\pi = 0 \Rightarrow q = (\sigma - 1)\varphi f$
- Labor market equilibrium: $n\left(f + \frac{q}{\varphi}\right) = L \Rightarrow n = \frac{L}{\sigma f}, n^* = \frac{L^*}{\sigma f}$

▸ Price indices, wages

Predictions of the Krugman Model

- There is intra-industry trade even if countries are identical, so long as they produce different varieties.
- Prices indices are lower than in autarky, and therefore welfare is higher.
- There are more firms in the large country.
- Price indices are equal when $\tau = 1$, but lower in the large country when $\tau > 1$. Fewer varieties bear a transport cost (see Appendix).
- Wages are higher in the large country, which guarantees trade balance (see Appendix).

From Theory to Gravity Regressions

- Value of aggregate exports: $X = n\tau p q^X(\tau p)$

  with:

$$
\begin{aligned}
q^X(\tau p) &= \left(\frac{\tau p}{P*}\right)^{-\sigma} \frac{w^* L^*}{P*} \\
p &= \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} \\
n &= \frac{L}{\sigma f}
\end{aligned}
$$

$$
\Rightarrow X = \frac{1}{\sigma f} \left(\frac{\sigma}{(\sigma - 1)\varphi}\right)^{1-\sigma} LL^* \left(\frac{\tau w}{P*}\right)^{1-\sigma} w^*
$$

or in log:

$$
\ln X = -\ln(\sigma f) + (1-\sigma)\ln\frac{\sigma}{(\sigma-1)\varphi} + \ln L + \ln L^* + (1-\sigma)\ln\frac{\tau w}{P*} + \ln w^*
$$

From Theory to Gravity Regressions (2)

- Gravity regressions (Tinbergen, 1962)
  Bilateral trade flows follow a 'gravity law' of the form

$$X_{ij} = G \frac{(L_i)^{\alpha}(L_j)^{\beta}}{(D_{ij})^{\theta}}$$

  L_i: size of country i; D_{ij}: distance between i and j.
- The Krugman model is consistent with that finding if $\alpha = \beta = 1$ and distance is a good proxy for bilateral transport costs τ (See PC).
- Transport costs affect trade along two 'margins':
  - increase in the number of available products (extensive margin)
  - increase in the value of export per product (intensive margin)

International Trade: The gravity equation

$$\ln X_{ij} = a + \ln L_i + \ln L_j + (1-\sigma)\ln \tau_{ij} + (1-\sigma)\ln w_i - (1-\sigma)\ln P_j + \ln w_j$$

	(1)	(2)	(3)	(4)	(5)	(6)
ln Pop, i	0.978[a]	0.893[a]	0.290[a]			
	(0.006)	(0.009)	(0.046)			
ln Pop, j	0.837[a]	0.835[a]	0.962[a]			
	(0.006)	(0.008)	(0.040)			
ln GDP/Pop, i	1.118[a]	0.921[a]	0.732[a]			
	(0.007)	(0.010)	(0.015)			
ln GDP/Pop, j	0.945[a]	0.702[a]	0.634[a]			
	(0.007)	(0.010)	(0.015)			
ln Dist (avg)	-1.035[a]	-1.197[a]				
	(0.014)	(0.015)				
Shared Language	0.506[a]	0.522[a]				
	(0.034)	(0.038)				
Shared Legal Origins	0.313[a]	0.160[a]				
	(0.026)	(0.029)				
Colonial History	1.560[a]	2.605[a]				
	(0.380)	(0.206)				
RTA	0.958[a]	0.593[a]	0.521[a]	0.400[a]	0.411[a]	0.317[a]
	(0.044)	(0.026)	(0.027)	(0.029)	(0.034)	(0.033)
Both GATT	0.125[a]	0.155[a]	0.159[a]	0.244[a]	0.368[a]	0.206[a]
	(0.020)	(0.016)	(0.017)	(0.038)	(0.041)	(0.042)
Currency union	0.688[a]	0.483[a]	0.486[a]	0.499[a]	0.469[a]	0.309[a]
	(0.091)	(0.064)	(0.068)	(0.047)	(0.056)	(0.089)
Tetrads:				GBR,FRA	USA,DEU	CHE,CAN
Fixed Effects:	None	Dyads(RE)	Dyads	Tetrads	Tetrads	Tetrads
# Obs.	618233	618233	618233	665531	651603	633190
RMSE	2.165	1.480	1.473	1.677	1.722	1.832

Source: Head, Mayer and Ries (2008)

Welfare gains

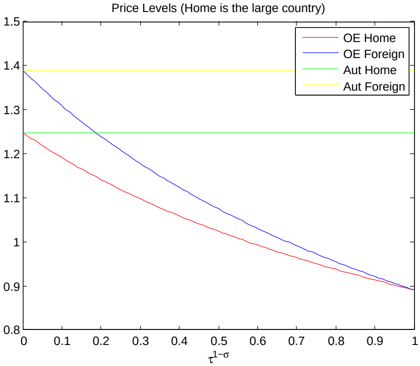
- Autarky: $P = pn^{\frac{1}{1-\sigma}}$ and $P^* = p^* {n^*}^{\frac{1}{1-\sigma}}$
- Open economy: $P = \left[p^{1-\sigma} n + (\tau p^*)^{1-\sigma} n^*\right]^{\frac{1}{1-\sigma}}$ and $P^* = \left[{p^*}^{1-\sigma} n^* + (\tau p)^{1-\sigma} n\right]^{\frac{1}{1-\sigma}}$
- Without transportation costs: $P = P^* = (2np^{1-\sigma})^{\frac{1}{1-\sigma}} < (np^{1-\sigma})^{\frac{1}{1-\sigma}}$ since $\sigma > 1$

$\Rightarrow$ **Opening up the economy yields a welfare gain deriving from more diversity.**

- In Krugman (1979), trade has a pro-competitive effect too (fall in p due to a rise in σ).

Welfare Gains (2)

Prices as a function of the "freeness" of trade $\tau^{1-\sigma}$



Wages

Trade Balance:

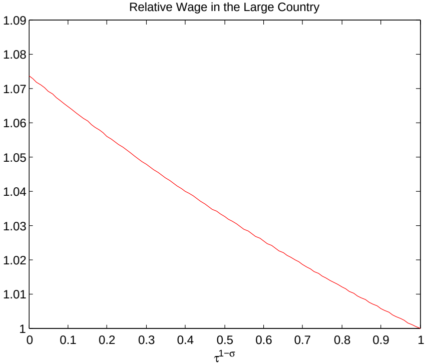
$$\underbrace{\lambda \times L \times L^* \times \left(\frac{\tau w}{P^*}\right)^{1-\sigma} \times w^*}_{X} = \underbrace{\lambda \times L \times L^* \times \left(\frac{\tau w^*}{P}\right)^{1-\sigma} \times w}_{X^*}$$

$$\Rightarrow \frac{w}{w^*} = \left(\frac{L w^{1-\sigma} + L^*(\tau w^*)^{1-\sigma}}{L(\tau w)^{1-\sigma} + L^* w^{*\,1-\sigma}}\right)^{1/\sigma}$$

- $\Rightarrow$ Without transport costs ($\tau = 1$), wages are equalized across countries
- $\Rightarrow$ With high transport costs ($\tau \to \infty$): $\frac{w}{w^*} \to \left(\frac{L}{L^*}\right)^{\frac{1}{2\sigma-1}}$, ie wages are higher in the largest country

Wages (2)

Relative wage in the large country, as a function of the "freeness" of trade $\tau^{1-\sigma}$



Conclusions

- The model generates gains from trade resulting from increased product diversity.
- **Absent transport costs**, all consumers have access to all varieties, prices converge and trade is balanced.
- With a transport cost, **the large country has lower prices** (if $L > L^*$, $P < P^*$) and more varieties. Demand for imports is lower (increasing with P).
- Balanced trade requires lower exports of the large country, thanks to a **higher marginal cost**: $w > w^*$ ▸ Proof
- Extension with mobile workers: migration towards the large country makes it larger... This is the foundation of the **'new economic geography' (Krugman 1991)**.

Appendix

How to derive the demand function

- Lagrangian: $L = \left(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} - \mu\left(\int_\Omega p(\omega)q(\omega)d\omega - wL\right)$
- First order conditions:

$$\begin{aligned} \frac{\partial L}{\partial q(\omega)} &= q(\omega)^{\frac{-1}{\sigma}}\left(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{1}{\sigma-1}} - \mu p(\omega) = 0 \\ \Leftrightarrow \quad & q(\omega)^{\frac{-1}{\sigma}} U^{\frac{1}{\sigma}} = \mu p(\omega) \\ \Leftrightarrow \quad & q(\omega) = U\mu^{-\sigma} p(\omega)^{-\sigma} \end{aligned}$$

- Budget constraint: $wL = \int_\Omega p(\omega)q(\omega)d\omega = U\mu^{-\sigma}\int_\Omega p(\omega)^{1-\sigma} d\omega$
- Define the price index $P = \left(\int_\Omega p(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}$. The budget constraint is rewritten as

$$wL = U\mu^{-\sigma} P^{1-\sigma}$$

- Note that $wL = PU$. P is the monetary value of one unit of utility.

How to derive the demand function (2)

- From the budget constraint $wL = U\mu^{-\sigma}P^{1-\sigma}$ and the f.o.c. $q(\omega) = U\mu^{-\sigma}p(\omega)^{-\sigma}$, one obtains the demand function:

$$q(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\sigma}\frac{wL}{P}$$

- Demand for variety ω increases with overall purchasing power wL/P, and decreases with the relative price of variety ω.
- σ is equal to
  - the price elasticity: a 1% rise in $p(\omega)$ reduces demand by σ%
  - the elasticity of substitution: since $\frac{q(\omega)}{q(\omega')} = \left(\frac{p(\omega)}{p(\omega')}\right)^{-\sigma}$, increasing the relative price of the ω variety by 1% reduces the relative demand for this variety by σ%

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How to derive the optimal price

- Start from the firm's profit function:

$$\pi(\omega) = p(\omega)q(\omega) - w\left(f + \frac{q(\omega)}{\varphi}\right)$$

- Maximize with respect to price given demand and price index P

⇒ First order condition:

$$\frac{\partial \pi(\omega)}{\partial p(\omega)} = P^{\sigma-1} wL \left[(1-\sigma)p^{-\sigma} + \frac{w}{\varphi}\sigma p^{-\sigma-1}\right] = 0$$

Or after rearranging:

$$p = \underbrace{\frac{\sigma}{\sigma-1}}_{Mark-up} \underbrace{\frac{w}{\varphi}}_{Marginal\ cost}$$

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Price indices in a two-country world economy

- The price index now writes:

$$P = \left(\sum_{\omega \in H} p(\omega)^{1-\sigma} + \sum_{\omega \in F} (\tau p^*(\omega))^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

- In the symmetric equilibrium, $p(\omega) = p$, $\forall \omega \in H$ and $p^*(\omega) = p^*$, $\forall \omega \in F$

$$\Rightarrow \quad P = \left(np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

$$and \quad P^* = \left(n(\tau p)^{1-\sigma} + n^* p^{*\,1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

- Absent transportation costs ($\tau = 1$), with identical countries, the two indices are equal: $P = P^* = (2n)^{\frac{1}{1-\sigma}} p$. Both countries have access to the same varieties in the same conditions.
- Both indices are lower than those in autarky, which are: $P = n^{\frac{1}{1-\sigma}} p$ and $P^* = n^{\frac{1}{1-\sigma}} p^*$
- At given wages, opening up the economy has a positive impact on welfare ($U = wL/P$). This comes from consumers' **preference for diversity**

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Wages in the Krugman model

- We have expressed optimal prices $p(\omega)$ and P as functions of nominal income wL.
- L is exogenous but w is endogenous
- The wage is determined by the goods market equilibrium equation. Due to Walras' law, it is equivalent to rely on (i) the domestic market ($wL = \sum_{\omega} wl(\omega)$); (ii) the foreign market ($w^* L^* = \sum_{\omega} w^* l^*(\omega)$); (iii) the trade balance ($X = X^*$)
- We use the trade balance:

$$\lambda \times L \times L^* \times \left(\frac{\tau w}{P^*}\right)^{1-\sigma} \times w^* = \lambda \times L \times L^* \times \left(\frac{\tau w^*}{P}\right)^{1-\sigma} \times w$$

$$\Rightarrow \quad \frac{w}{w^*} = \left(\frac{P}{P^*}\right)^{\frac{1-\sigma}{\sigma}}$$

$$\textit{with} \quad \left(\frac{P}{P^*}\right) = \frac{np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma}}{n(\tau p)^{1-\sigma} + n^* p^{*\,1-\sigma}}$$

$$\Rightarrow \quad \frac{w}{w^*} = \left(\frac{Lw^{1-\sigma} + L^*(\tau w^*)^{1-\sigma}}{L(\tau w)^{1-\sigma} + L^* w^{*\,1-\sigma}}\right)^{1/\sigma}$$

Wages in the Krugman model (2)

Relative imports:

$$\frac{M}{M^*} = \frac{n^*}{n}\left(\frac{\tau p^*/P}{\tau p/P^*}\right)^{1-\sigma}\frac{wL}{w^*L^*} = \frac{w}{w^*}\left(\frac{w^*/P}{w/P^*}\right)^{1-\sigma}$$

- Starting from the symetric equilibrium: $\frac{L}{L^*} = \frac{w}{w^*} = \frac{M}{M^*}$
- An increase in L/L^* increases the relative number of firms in H which reduces P/P^*. This makes foreign goods relatively more expansive $\rightarrow \downarrow M/M^*$
- For trade to be balanced, needs to be compensated by an increase in the relative wage $w/w^* \rightarrow \uparrow M/M^*$ through an income effect ($\uparrow$ aggregate demand) and a substitution effect ($\downarrow$ relative competitiveness of domestically produced varieties)

$\Rightarrow$ Wages are relatively high in large countries Back to section 1