# III - Factor Proportions: the Heckscher-Ohlin Model

- **Differences in *factor abundance*, not technology, create gains from trade.**
- **Supply side: perfect competition on product and factor markets**
- **Demand side: identical, homothetic preferences**
- **We will focus on the 2 countries x 2 products x 2 factors case. Extensions will be discussed in the conclusion.**

- **1. The Closed Economy**
- 2 goods, X and Y
- 2 inputs, labor, L, and capital, K
  - immobile across countries, perfectly mobile across sectors
- Factor endowments: $\overline{L}$ and $\overline{K}$
- Technology: *constant* returns to scale

$$\begin{cases} X = F_X(L_X, K_X) \\ Y = F_Y(L_Y, K_Y) \end{cases} \text{ subject to } \begin{cases} L_X + L_Y \leq \overline{L} \\ K_X + K_Y \leq \overline{K} \end{cases}$$

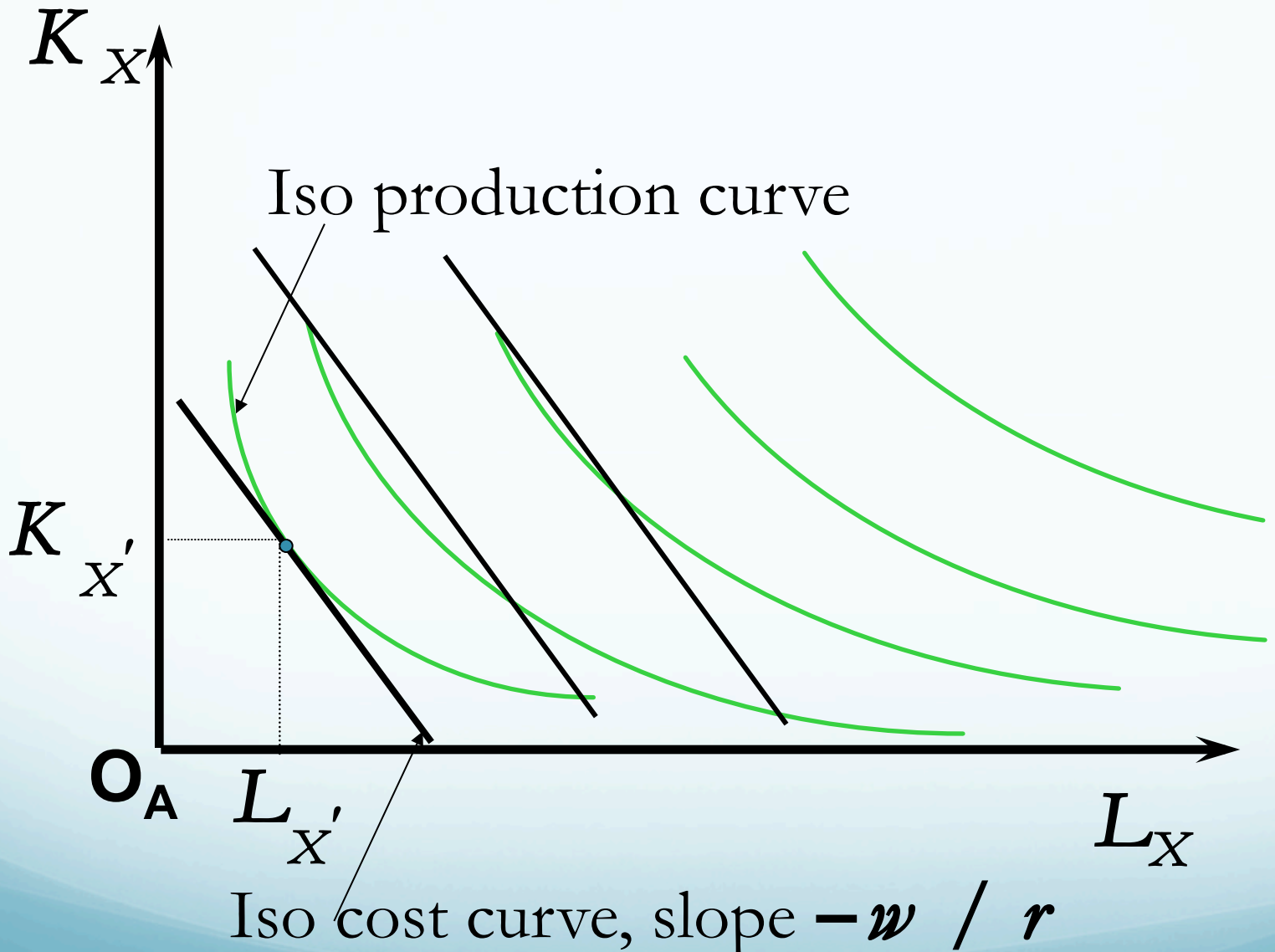
with decreasing marginal productivities

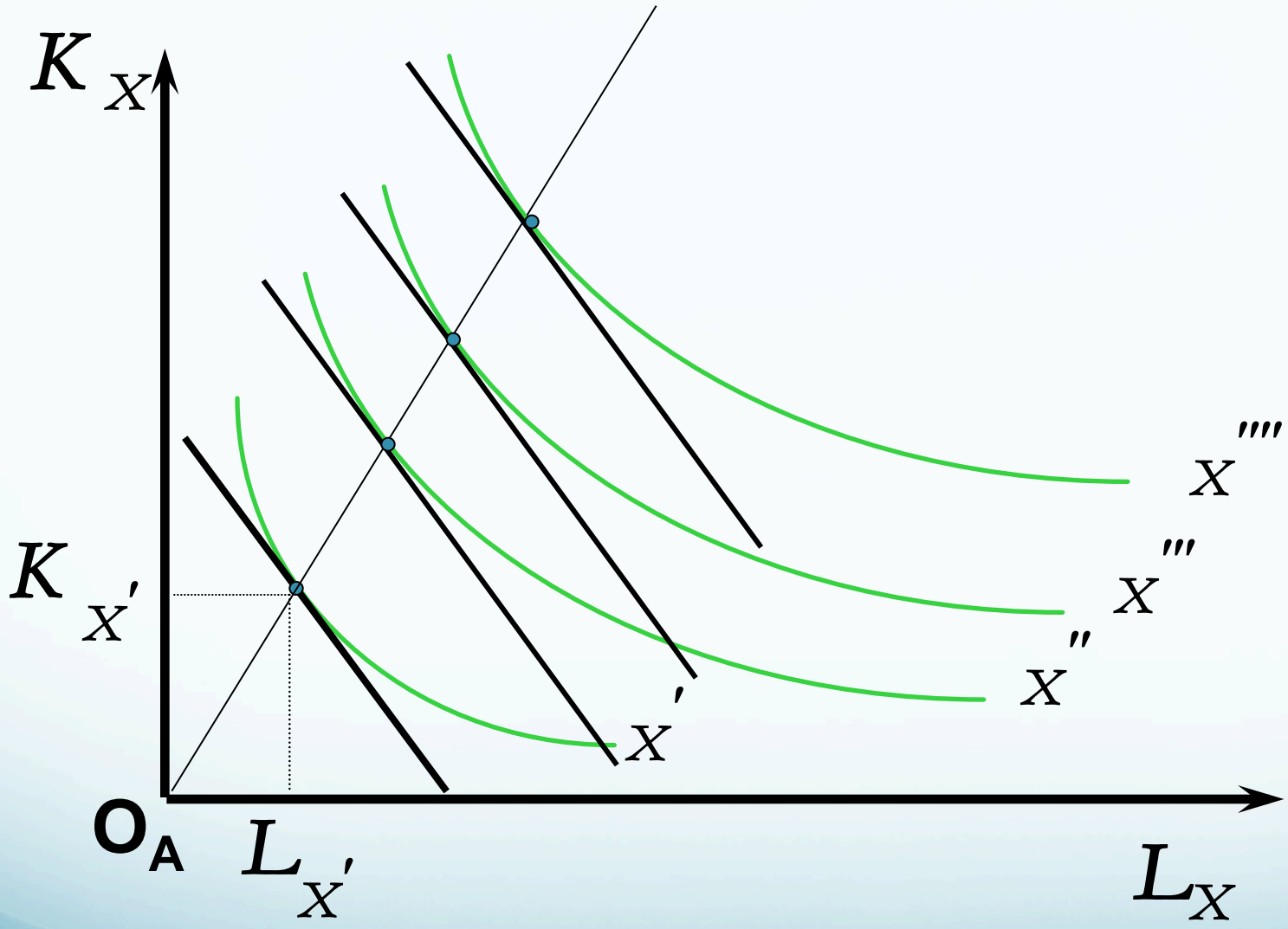
## Cost function

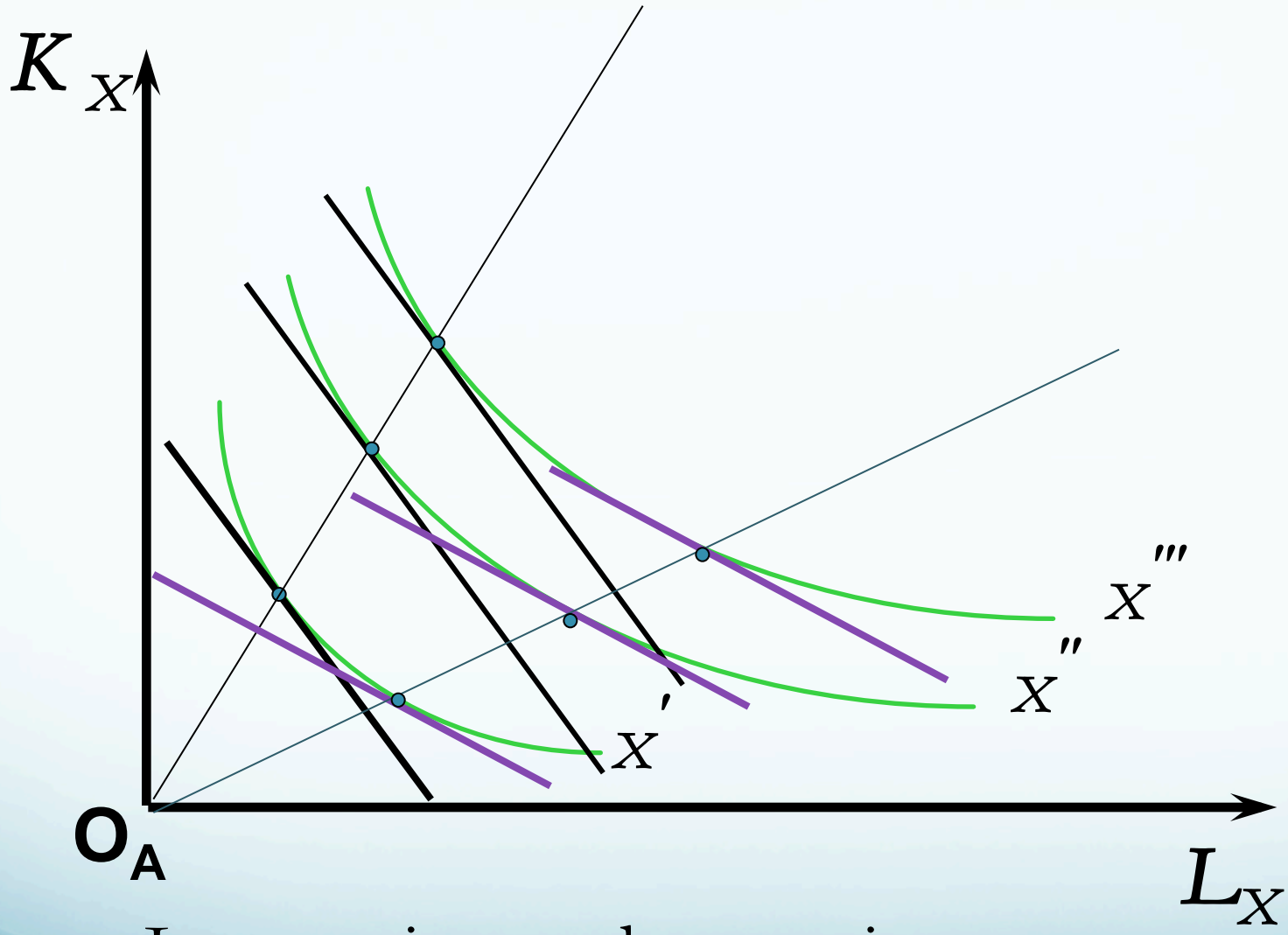
and

$$\begin{cases} \underset{L_X, K_X}{Min} \left( wL_X + rK_X \right) \\ st \; X = F_X \left( L_X, K_X \right) \end{cases}$$

$$\begin{cases} \underset{L_Y, K_Y}{Min} \left( wL_Y + rK_Y \right) \\ st \; Y = F_Y \left( L_Y, K_Y \right) \end{cases}$$


$K_X$
Iso production curve
$K_{X'}$
$O_A$
$L_{X'}$
$L_X$
Iso cost curve, slope $-w / r$


$K_X$
$K_{X'}$
$O_A$
$L_{X'}$
$L_X$
$X'$
$X''$
$X'''$
$X''''$




Increase in *r* or decrease in *w*

- **With CRS, at each scale of production,**
  - the capital labor ratio is fixed and function of factor prices only
  - the average and the marginal cost of production are independent of the quantities

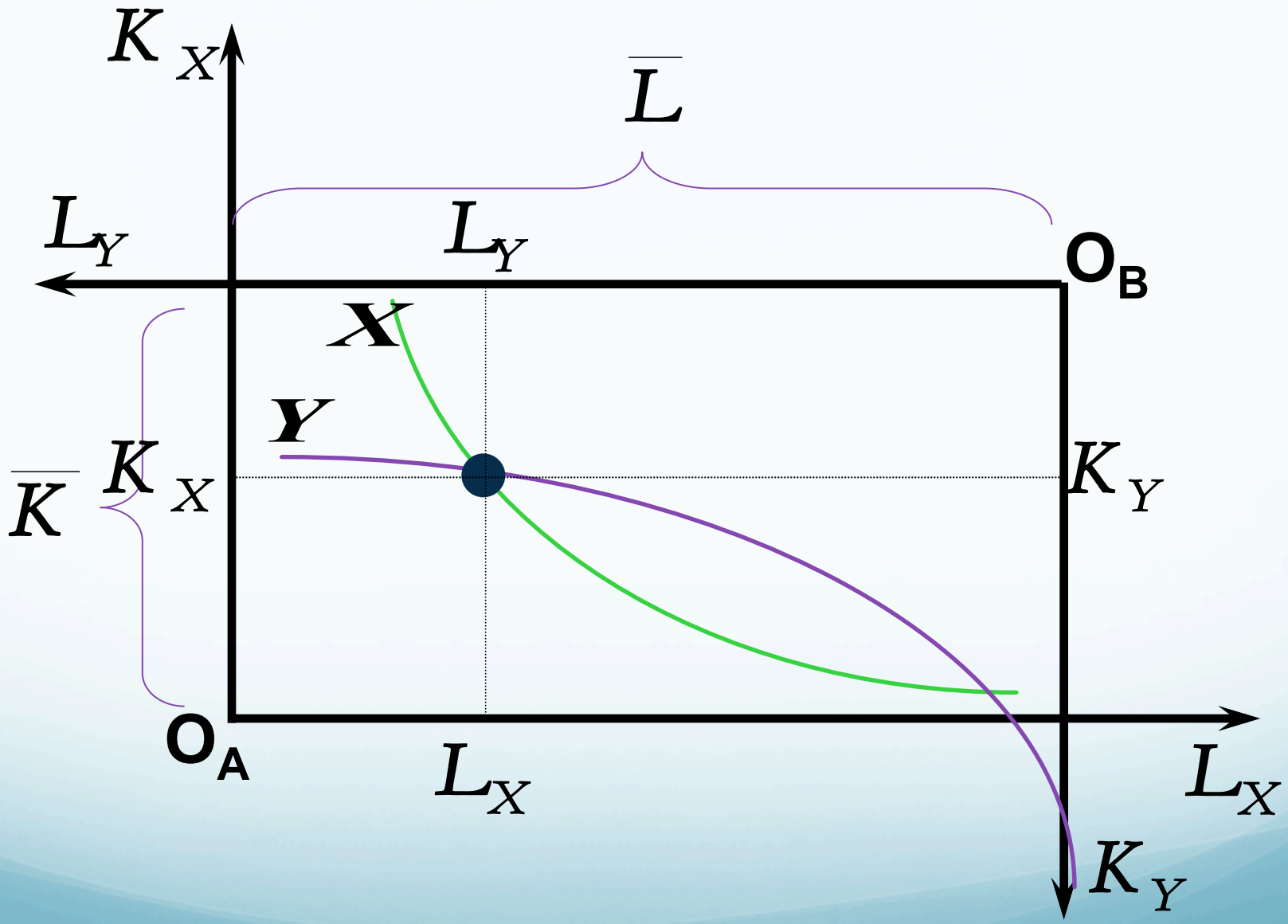
  they are function of factor prices only

  $\Rightarrow$

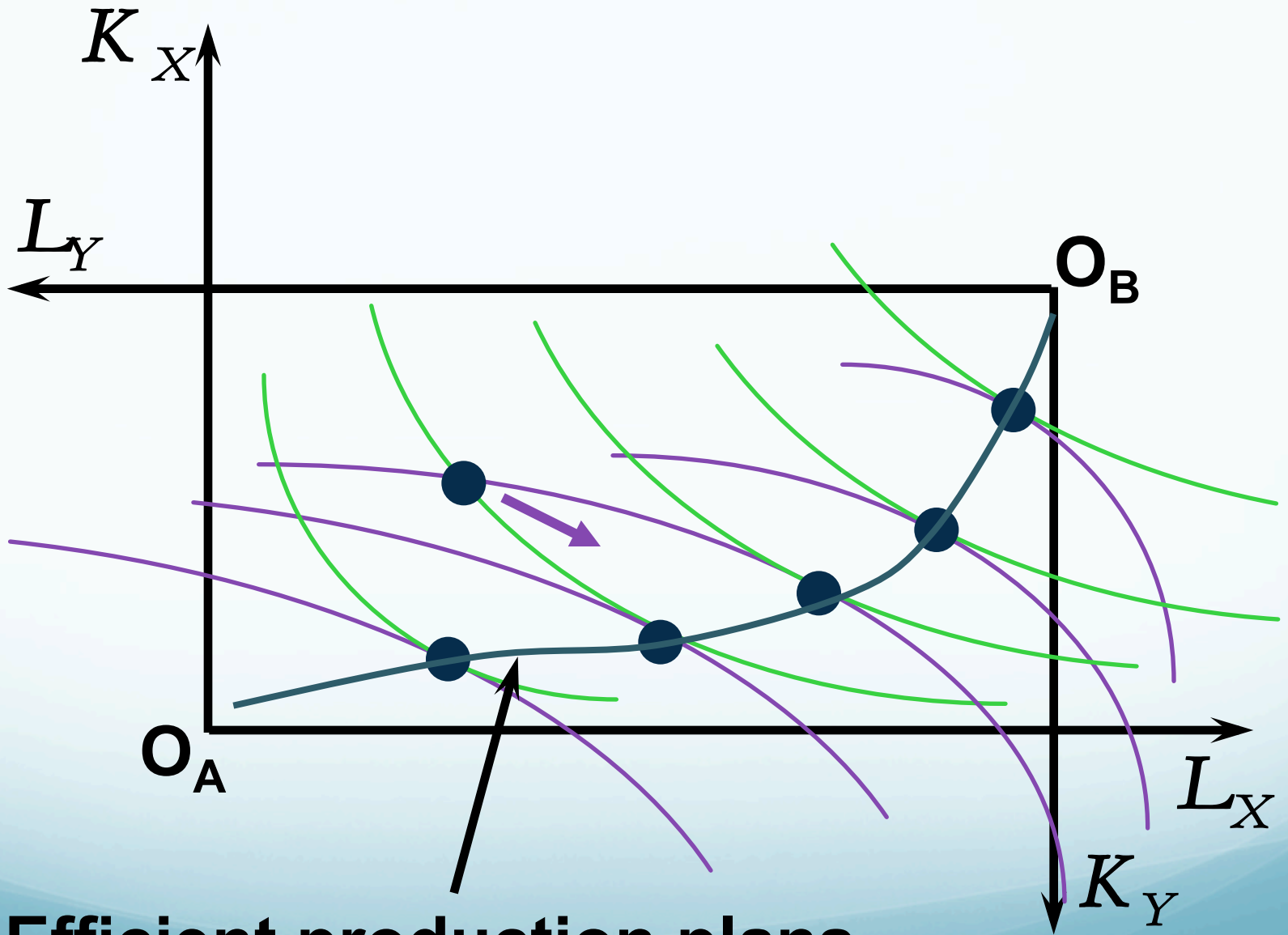
$$TC_X(X, w, r) = c_x(r, w)X$$

$$TC_Y(Y, w, r) = c_y(r, w)Y$$

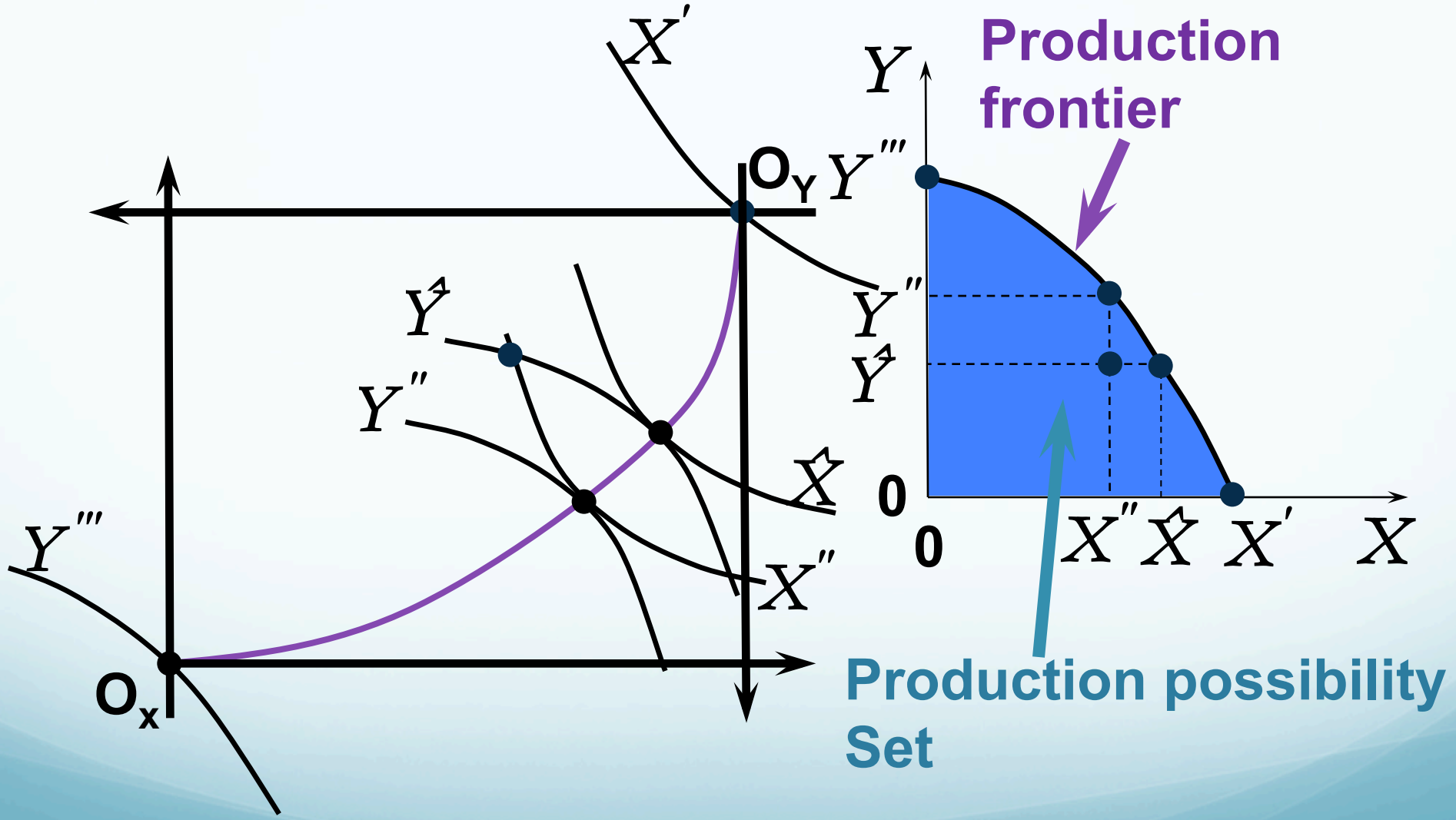
## Figure: Feasible production plans



- Figure: Efficient production plans



## Figure: Production possibilities



- Constant returns to scale assumption

  $\Rightarrow$ concave production frontier

- Production frontier slope:

$$\frac{dY}{dX} = -\frac{\partial F_Y / \partial L_Y}{\partial F_X / \partial L_X} = -\frac{\partial F_Y / \partial K_Y}{\partial F_X / \partial K_X}$$

- Profit maximisation and perfect competition imply that factors are priced at their marginal productivity i.e

$$\partial F_Y / \partial L_Y = w / p_y \quad and \quad \partial F_X / \partial L_X = w / p_x$$

$$\partial F_Y / \partial K_Y = r / p_y \quad and \quad \partial F_X / \partial K_X = r / p_x$$

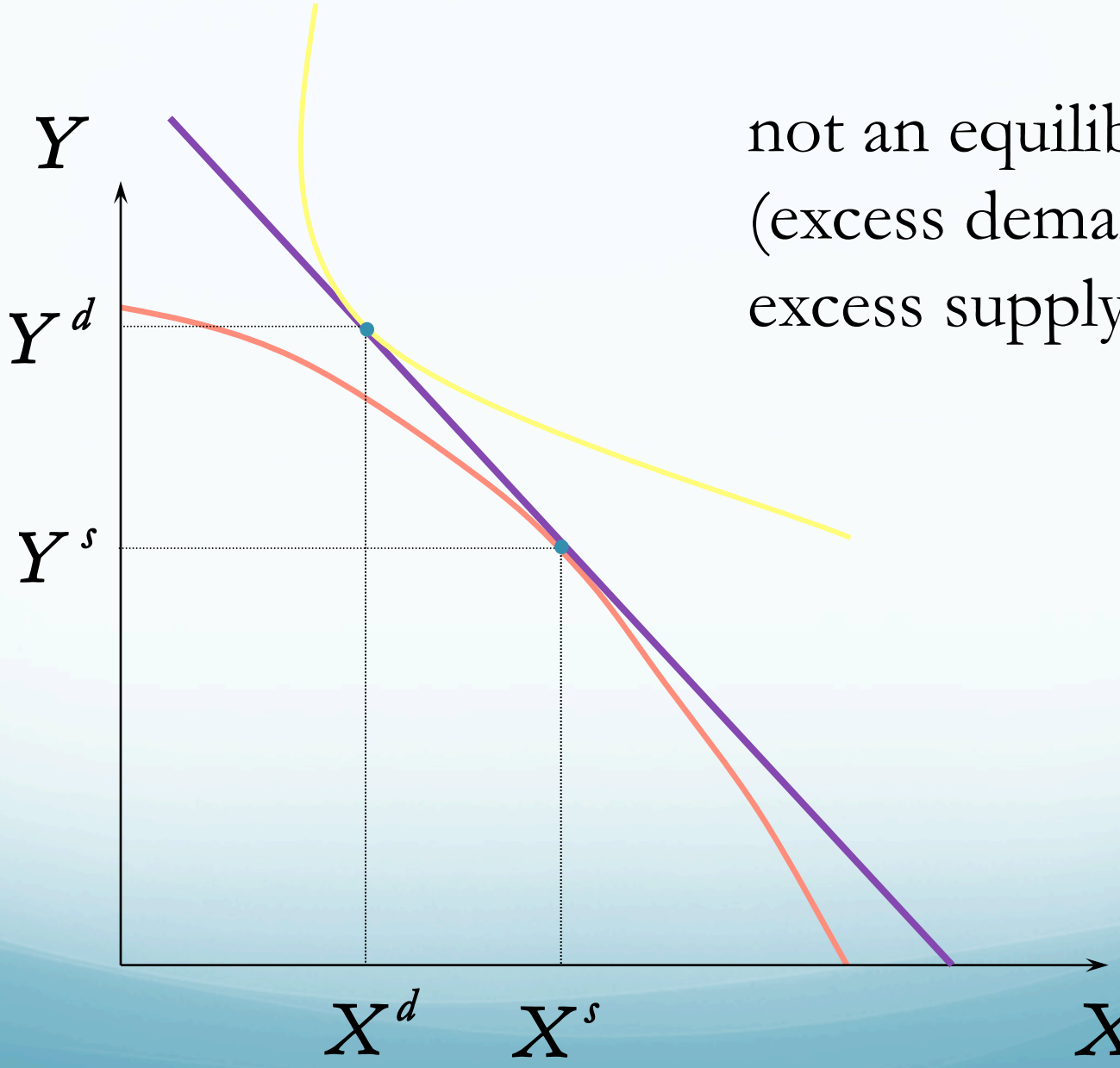
therefore:

$$\frac{dY}{dX} = -\frac{\partial F_Y / \partial L_Y}{\partial F_X / \partial L_X} = -\frac{\partial F_Y / \partial K_Y}{\partial F_X / \partial K_X} = -p_x / p_y$$

- Assumption: same preferences across consumers
- Competitive general equilibrium under autarky:

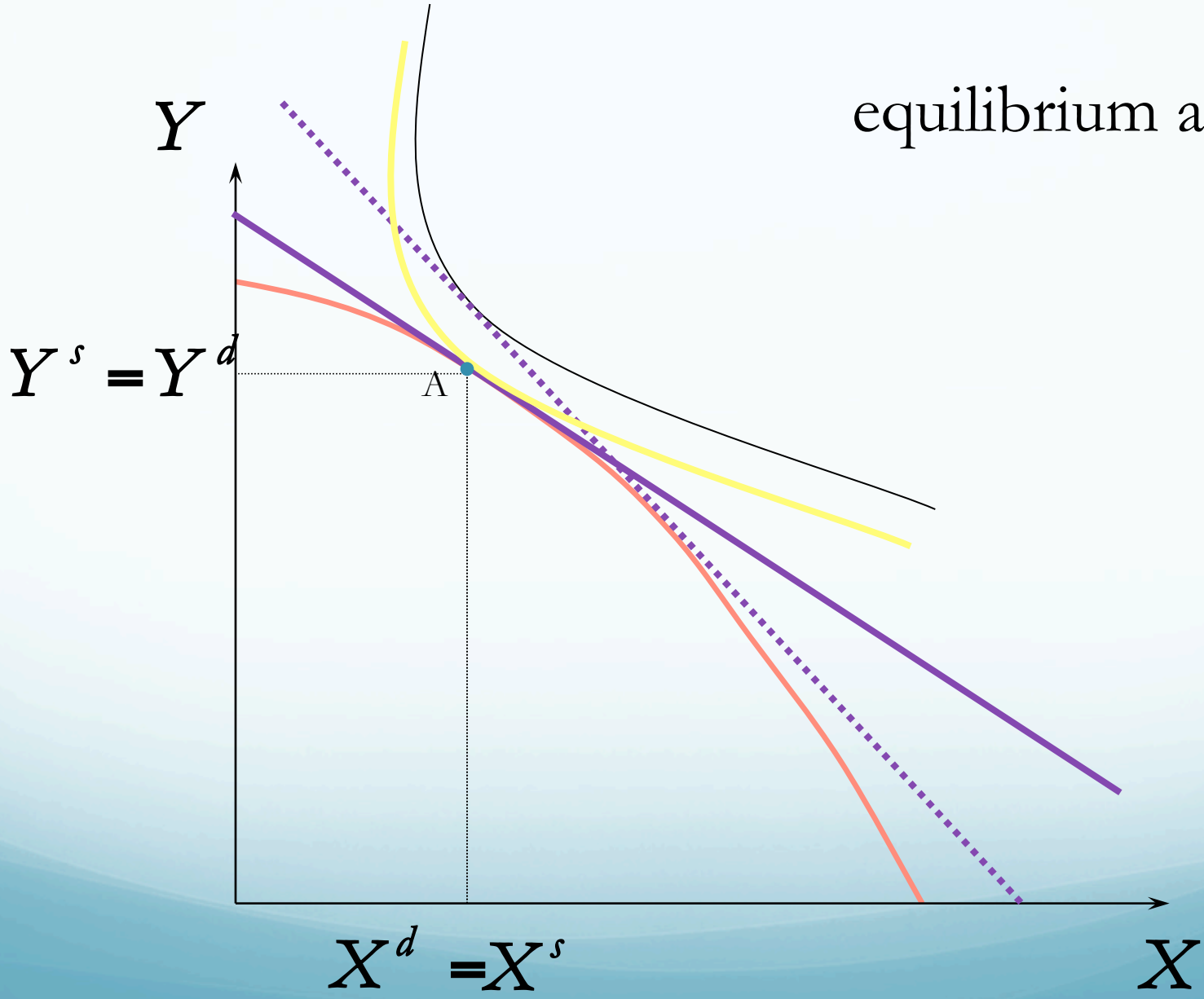
$$\begin{cases} p^i = MRS^i \text{ (consumer optimality)} \\ p^i = MRT^i \text{ (firm optimality)} \\ X^{i,d} = X^{i,s} \text{ and } Y^{i,d} = Y^{i,s} \text{ (market equilibrium)} \end{cases}$$

## Figure: Equilibrium under autarky




not an equilibrium
(excess demand of Y,
excess supply of X)

## Figure: Equilibrium under autarky

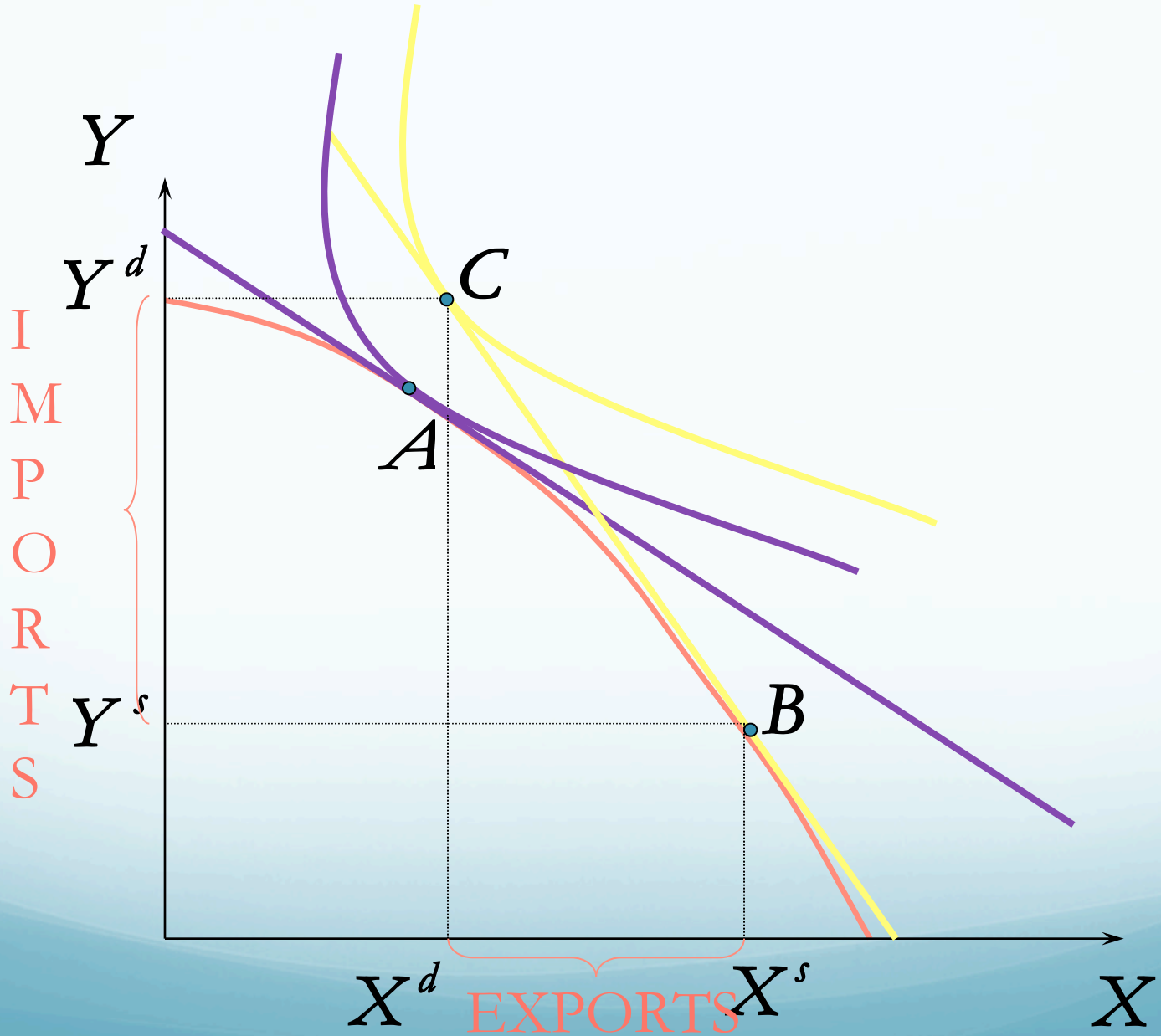


- **2. The Open Economy Equilibrium**
- **Firms and consumers face the world price,** $p^* = \frac{p_X^*}{p_Y^*}$
- **Competitive equilibrium characterization under open economy**

$$\begin{cases} p^* = MRS^i, \ \forall \ \text{i} \\ p^* = MRT^i, \ \forall \ \text{i} \\ p_X^* \left( X^{i,s} - X^{i,d} \right) + p_Y^* \left( Y^{i,s} - Y^{i,d} \right) = 0, \ \forall \ \ \text{i} \\ \sum_i X^{i,d} = \sum_i X^{i,s} \end{cases}$$

- Intuitions:
  - suppose $p^* > p^a$ There are incentives to produce more good **X**, and to consume more good **Y**, but possibility of incomplete specialization
  - if $p^* < p^a$ the opposite applies
  - if $p^* = p^a$ production and consumption are as in autarky

## Figure: Equilibrium under free trade



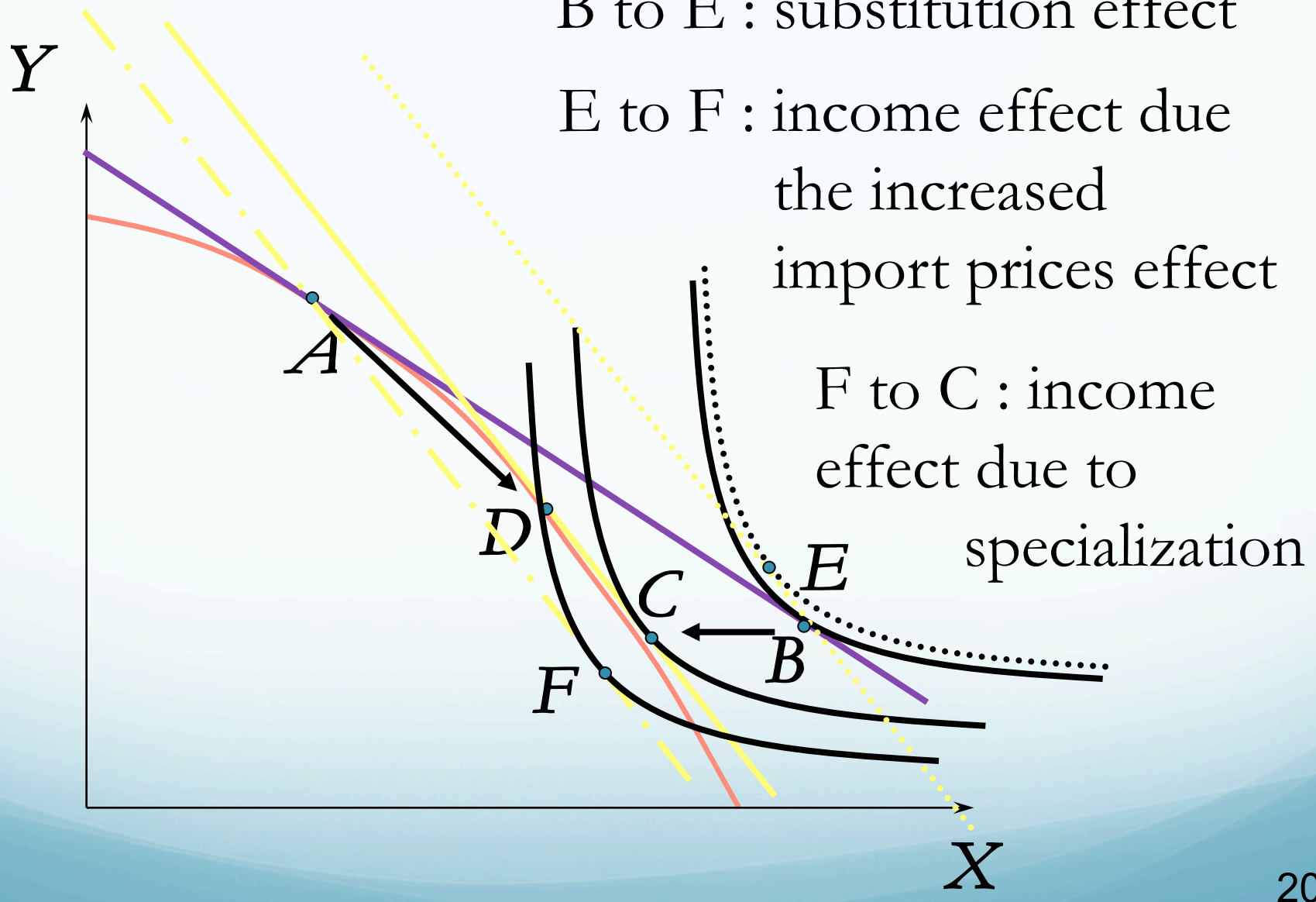
- **In the open economy, how does the excess demand function vary with p*?**
  - $X^s(p^*)$ **: clearly increasing (see the optimal production point on the production frontier)**
  - $X^d(p^*)$ **: several effects**

    - <u>Case 1</u>: When good X is imported**, if the world price increases:**

      ✓ **negative effect: substitution towards Y**

      ✓ **negative effect: income effect that is a combination of the loss due to the increase of the price of imports and some specialization effect**

    ⇒ **demand decreases unambiguously**



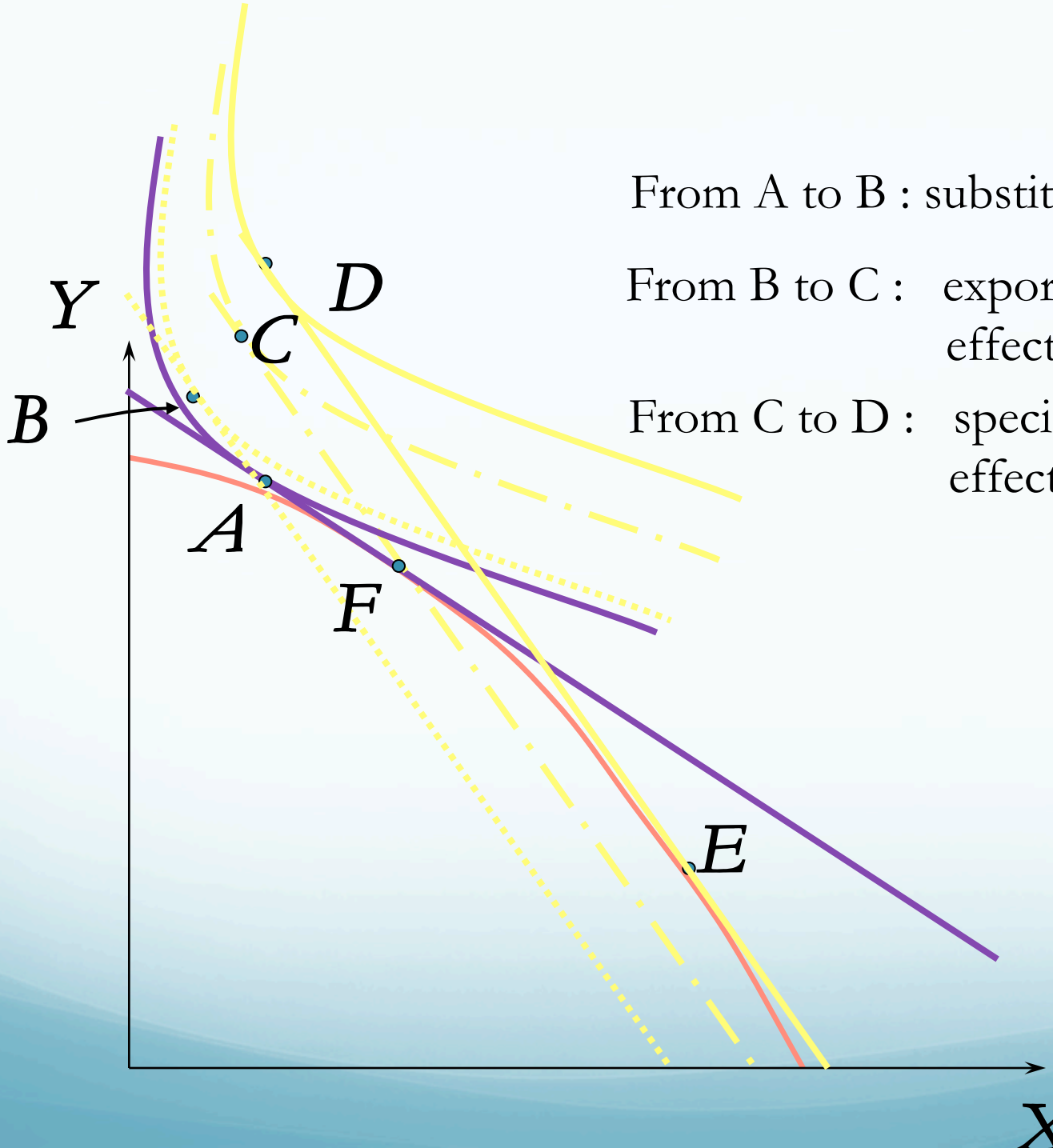

B to E : substitution effect

E to F : income effect due the increased import prices effect

F to C : income effect due to specialization

- **<u>Case 2</u>:** when good X is exported**, if the world price increases:**
  - ✓ **negative effect: substitution towards Y**
  - ✓ **positive effect: income effect due to the price increase of exports and some specialization effects**

⇒ **demand should decrease, but may increase if the last two effects are strong**

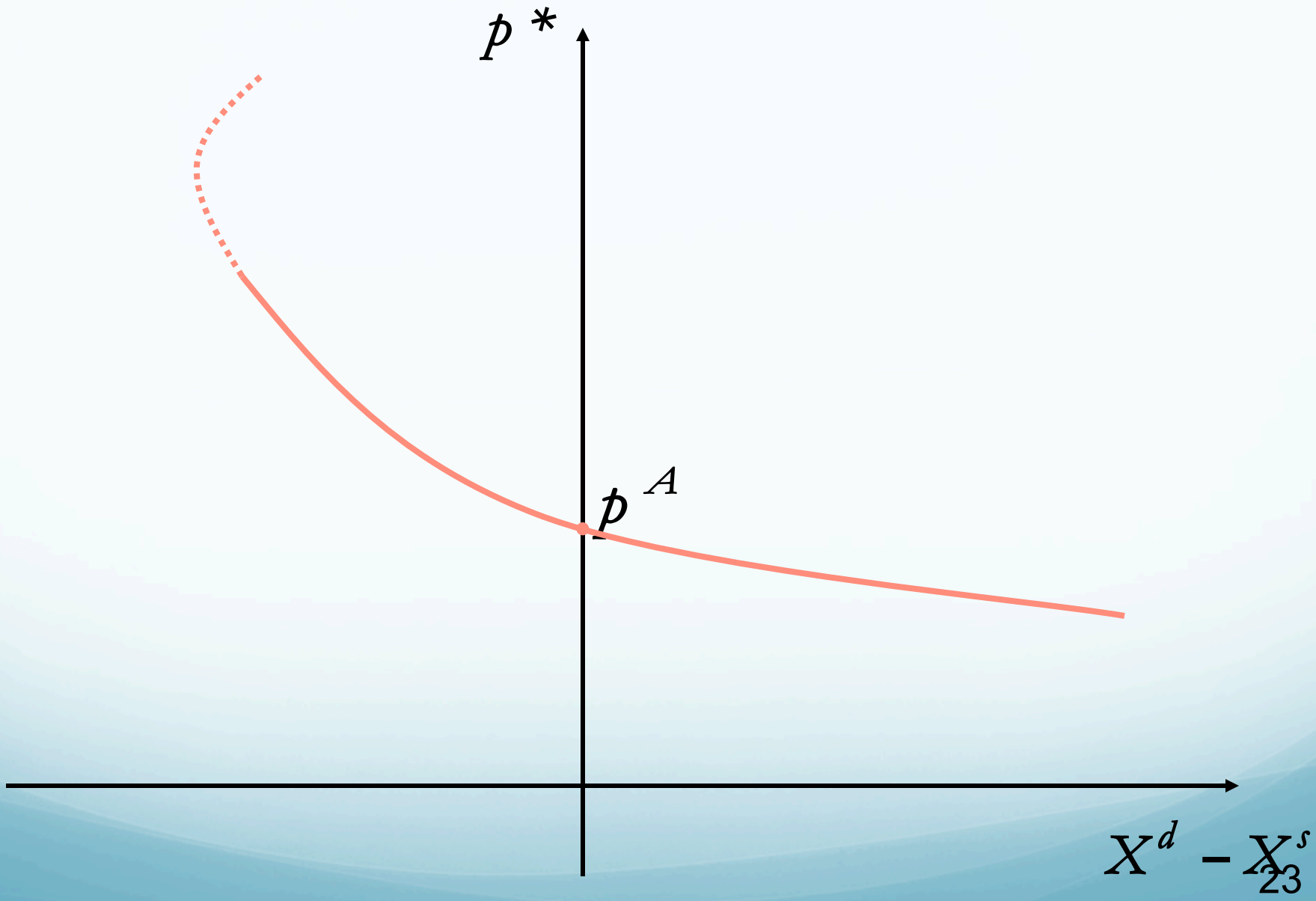



From A to B : substitution effect

From B to C : export price income effect

From C to D : specialisation income effect

## ⇒ Figure: Excess demand function plot



- Assumption: monotonicity of the excess demand function

  ⇒ unique world price (see below)

- World price
  - without loss of generality we assume $p_1^a \leq p_2^a$

    if $p_1^a = p_2^a$ , free trade changes nothing

    if $p_1^a < p_2^a$:

    ✓ world market equilibrium:

$$\Rightarrow \quad X_1^d + X_2^d = X_1^s + X_2^s \quad \Rightarrow$$

$$X_1^d - X_1^s + X_2^d - X_2^s = 0$$

(⇒ $Y_1^d + Y_2^d = Y_1^s + Y_2^s$ by using the budget constraint)

$$E_1 + E_2 = 0$$

✓ if $p^* < p_1^a$ , both countries would like to export good Y

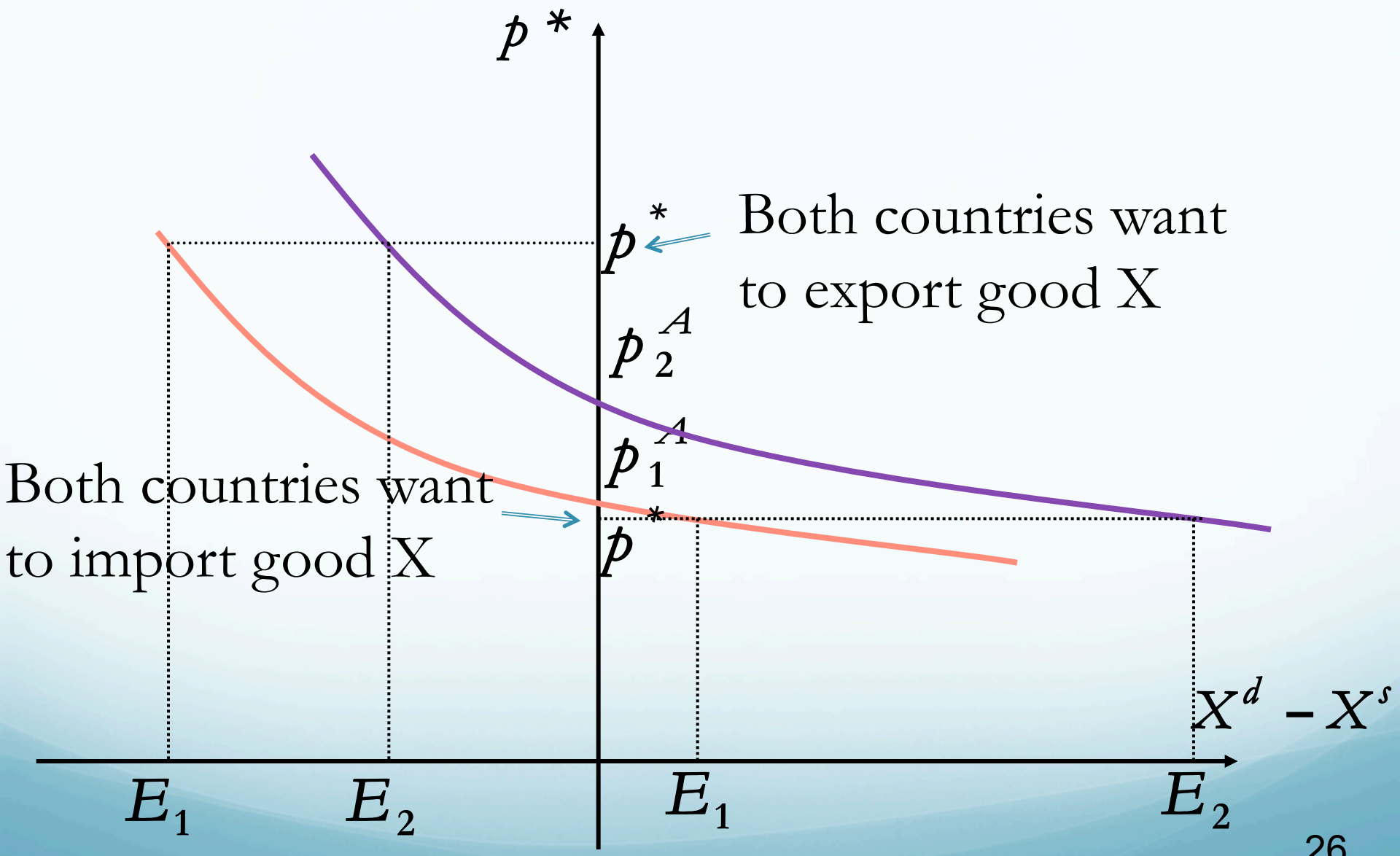
$\Rightarrow$ impossible

✓ if $p^* > p_2^a$ , both countries would like to export good X

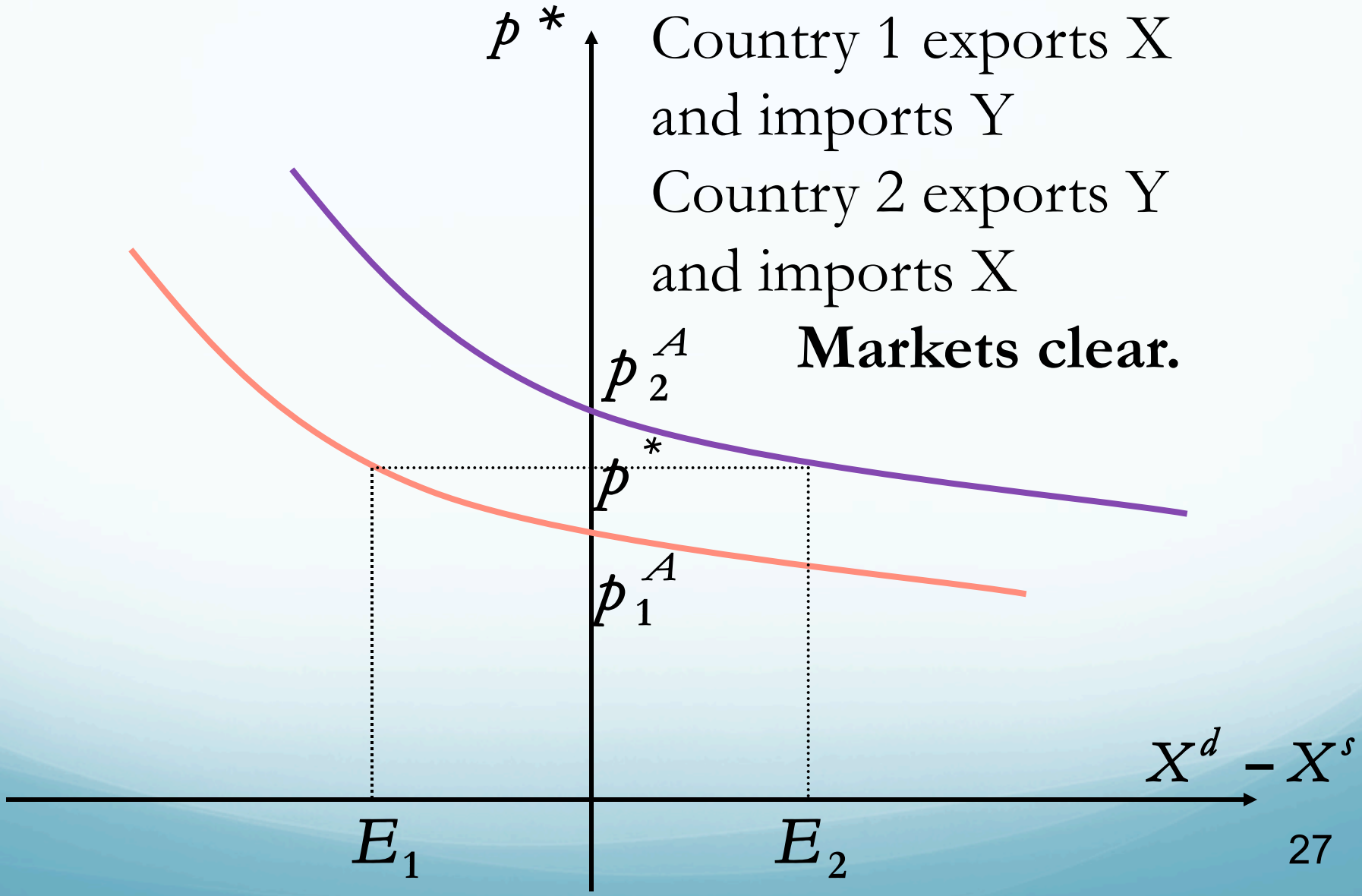
$\Rightarrow$ impossible

$\Rightarrow p^* \in \left[p_1^a, p_2^a\right]$ , such that $E_1 + E_2 = 0$

## Figure: Excess Demand and World Price



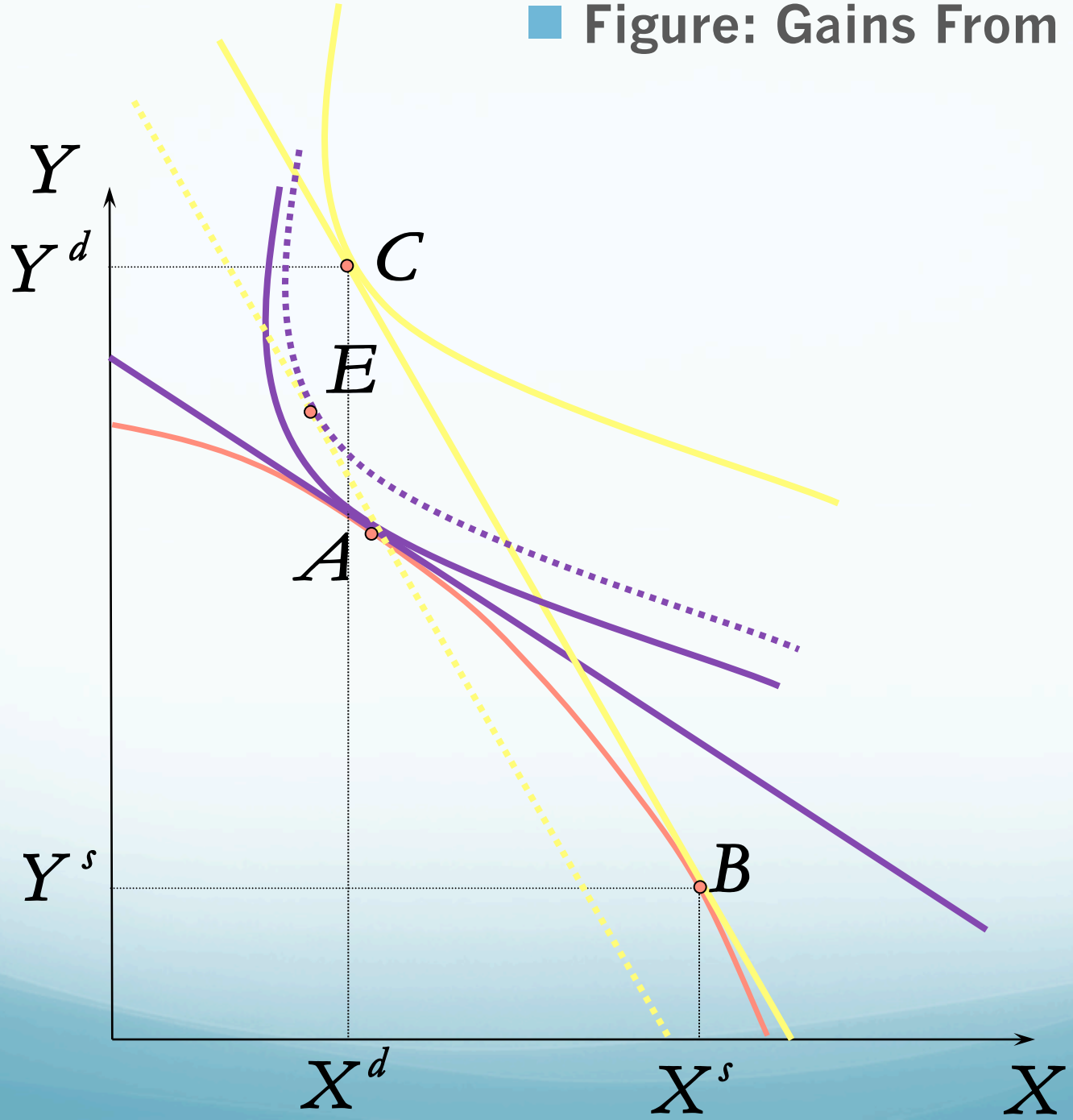
## Figure: Excess Demand Equalization and World Price



⇒ country 1 specializes (not completely) in good X and exports it

⇒ country 2 specializes (not completely) in good Y and exports it

## Figure: Gains From Trade



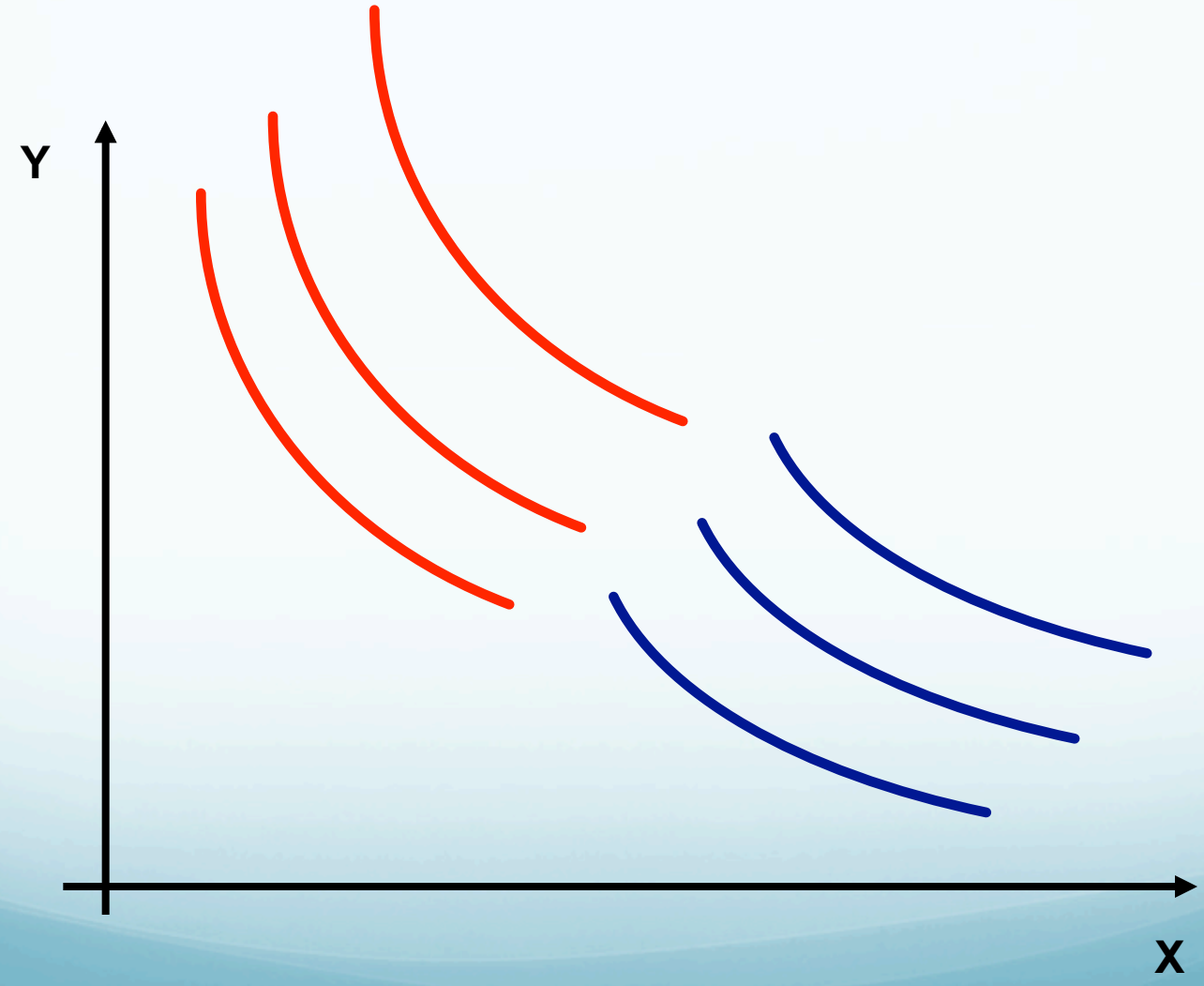
⇒ the representative consumer gains from free trade

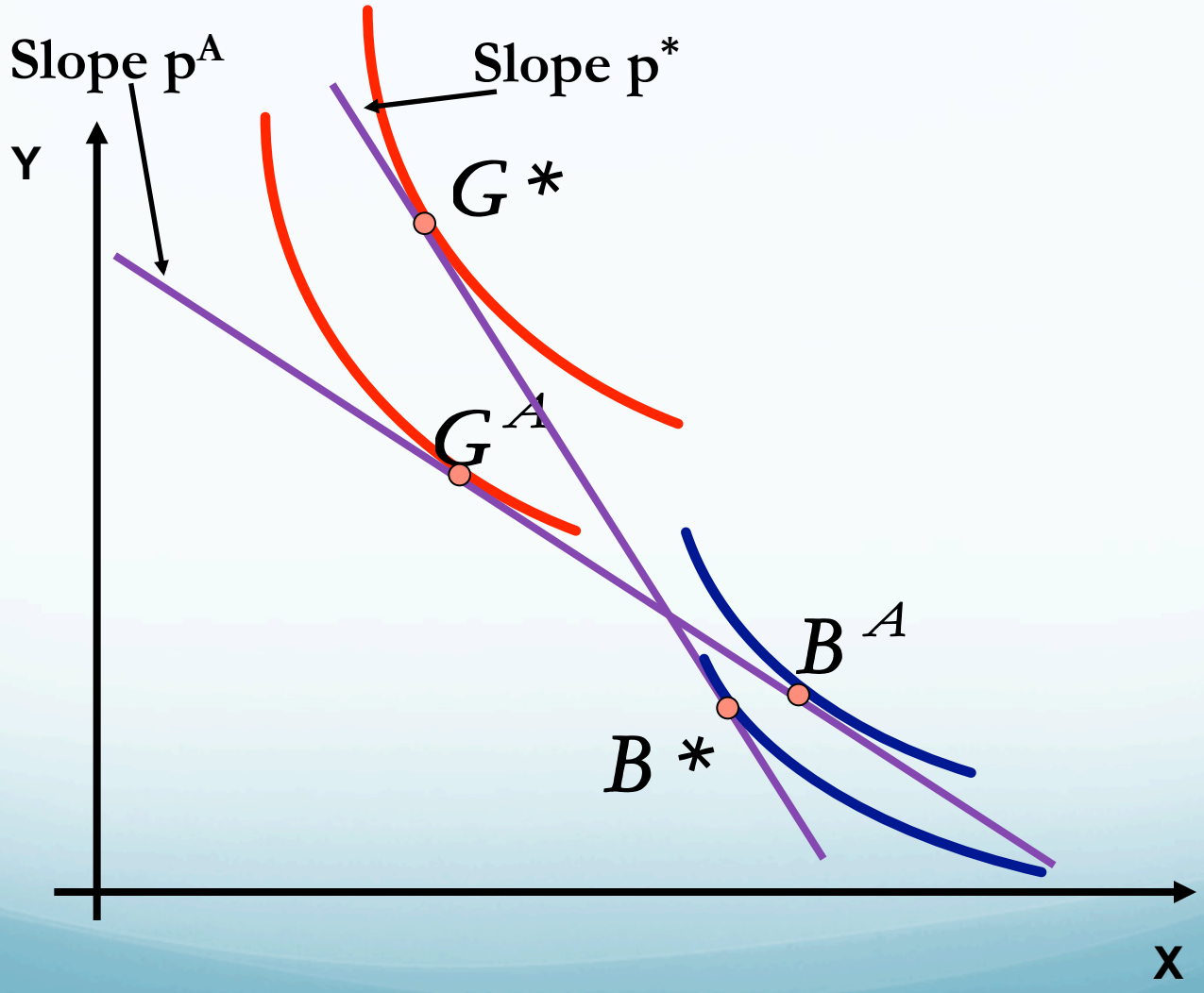
⇒ anyone who owns labor and capital in the same proportions as the country endowment gains from free trade

⇒ gains are the larger, the larger the price variation, the larger the differences between the countries

⇒ input are reallocated between sectors ⇒ are there net gains from trade if there are adjustment costs?

- **Difference in preferences inside a country: the case of 2 groups, G and B**




Slope p^A
Slope p*
Y
G *
G A
B A
B *
X

- intuitions:

✓ G consumers prefer good Y, B consumers prefer good X

✓ trade liberalization implies a decrease of the relative price of good Y

✓ G consumers gains more than average

✓ B consumers gains less than average and may lose

- BUT there always exists a way to redistribute that makes trade better than no trade

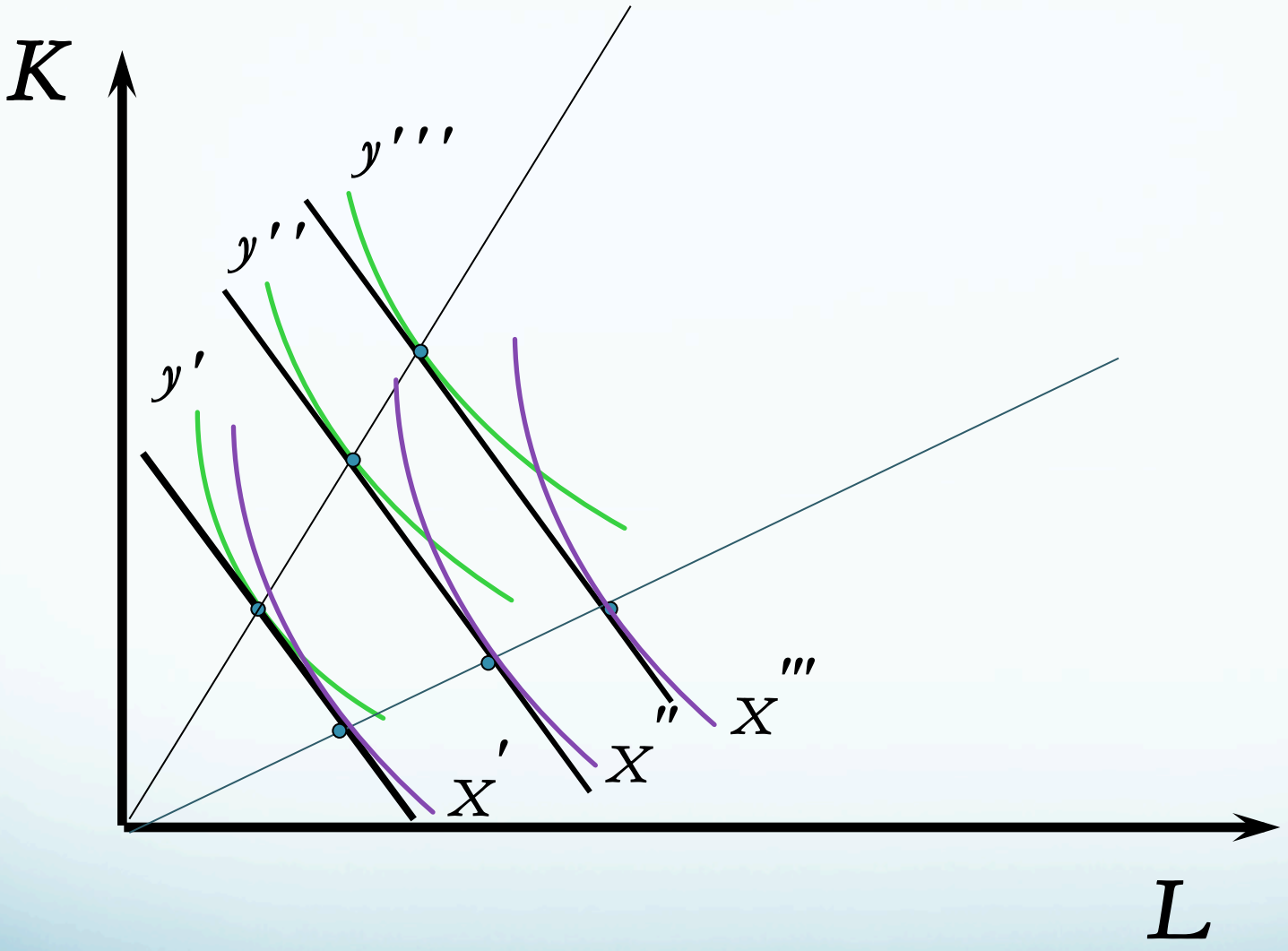
- **3. The Heckscher-Ohlin (HO) Theorem**
- The Heckscher-Ohlin model has 2 sectors with different factor intensities
- Definition: good Y is **relatively capital intensive** and good X is relatively labor intensive if the **capital-labor ratio** used in production is higher in sector Y at the production optimum

$$\Leftrightarrow \quad \frac{K_X}{L_X} < \frac{K_Y}{L_Y}$$

- **Assumption: no factor intensity reversal**

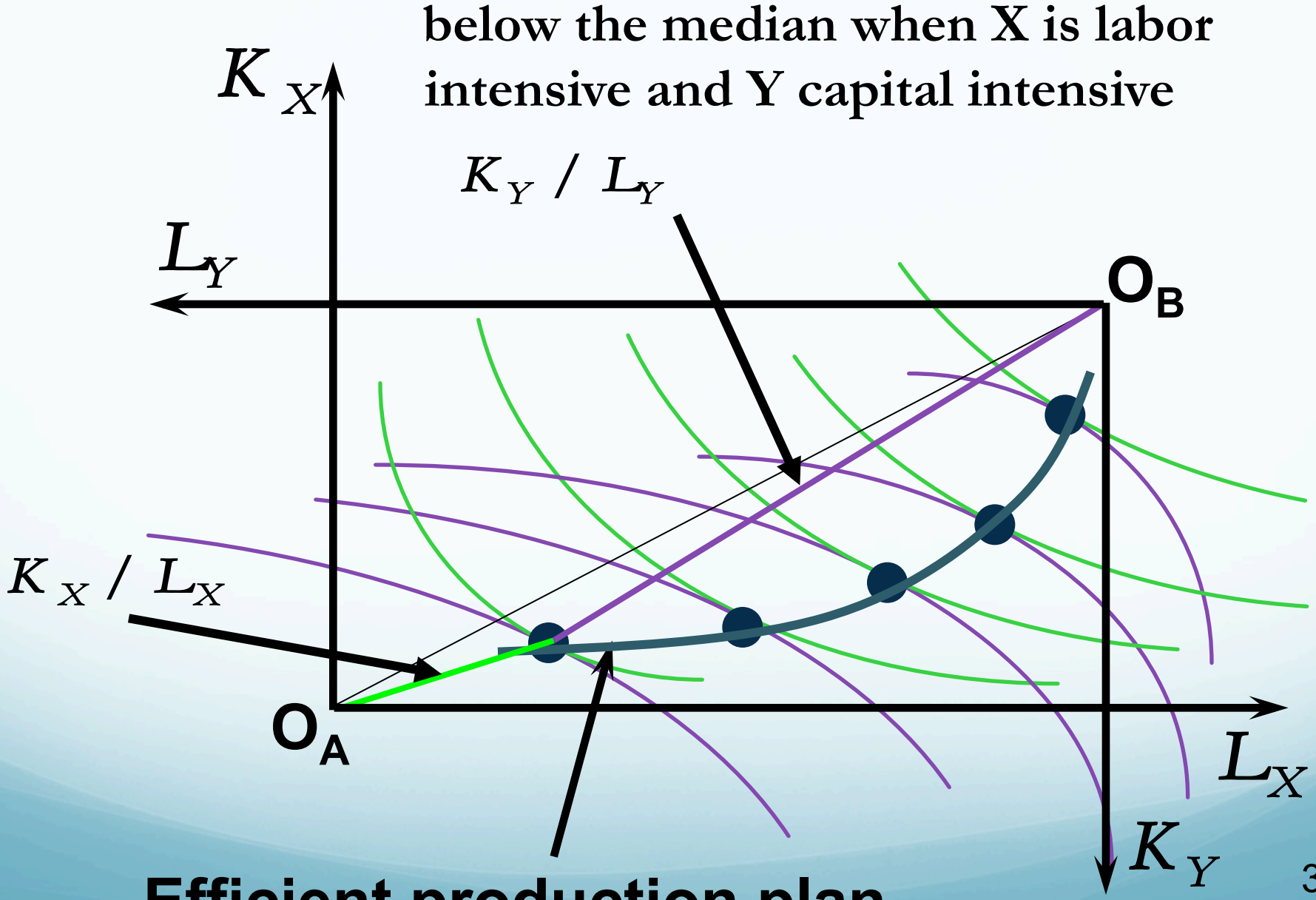
⇔ the ranking of factor intensities across sectors does not depend on the level of factor prices

⇔ $$\frac{K_X}{L_X} < \frac{K_Y}{L_Y} \quad \forall (r, w)$$




w and r are given and common across sectors

## Figure: Efficient production plan




**Efficient production plan**

- 2 countries with different relative factor endowments
- without loss of generality, we assume that country 1 is better endowed with capital

$$\left(\frac{\overline{K}}{\overline{L}}\right)_1 > \left(\frac{\overline{K}}{\overline{L}}\right)_2$$

- Note: *relative*, not absolute, differences in factor intensities and endowments matter

- **Table: Capital-labor ratios in selected US manufacturing industries in 1984**

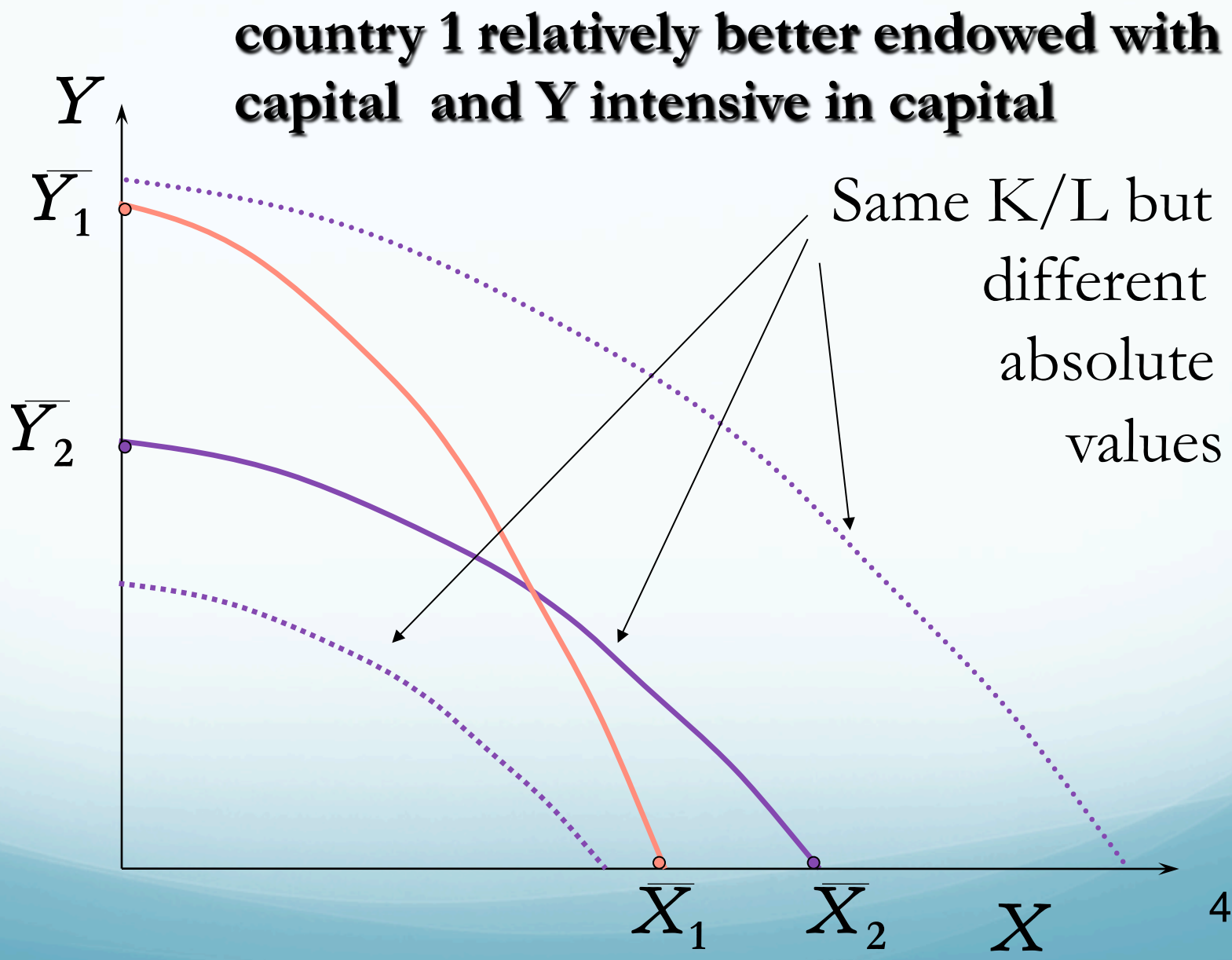
| Industry | K in $m | L (`000) | K/L |
|---|---|---|---|
| Petrol. refin. | 27 005 | 95 | 284 |
| Paper products | 33 007 | 613 | 53 |
| Iron and Steel | 25 607 | 505 | 50 |
| Transp. Equip. | 51 635 | 1 849 | 27 |
| Food prod. | 31 758 | 1 263 | 25 |
| Footwear | 514 | 107 | 4 |
| Wearing Apparel | 3 416 | 978 | 3 |

- **Table: Capital-labor endowments for selected countries in 1984**

| countries | K in $bn | L (m) | K/L $ |
|---|---|---|---|
| India | 482 | 254 | 1 898 |
| Brazil | 507 | 53 | 9 566 |
| Mexico | 353 | 23 | 15 348 |
| US | 3 696 | 116 | 32 421 |
| Canada | 419 | 12 | 34 917 |
| Germany | 1 018 | 26 | 39 154 |
| Japan | 2 336 | 59 | 39 593 |

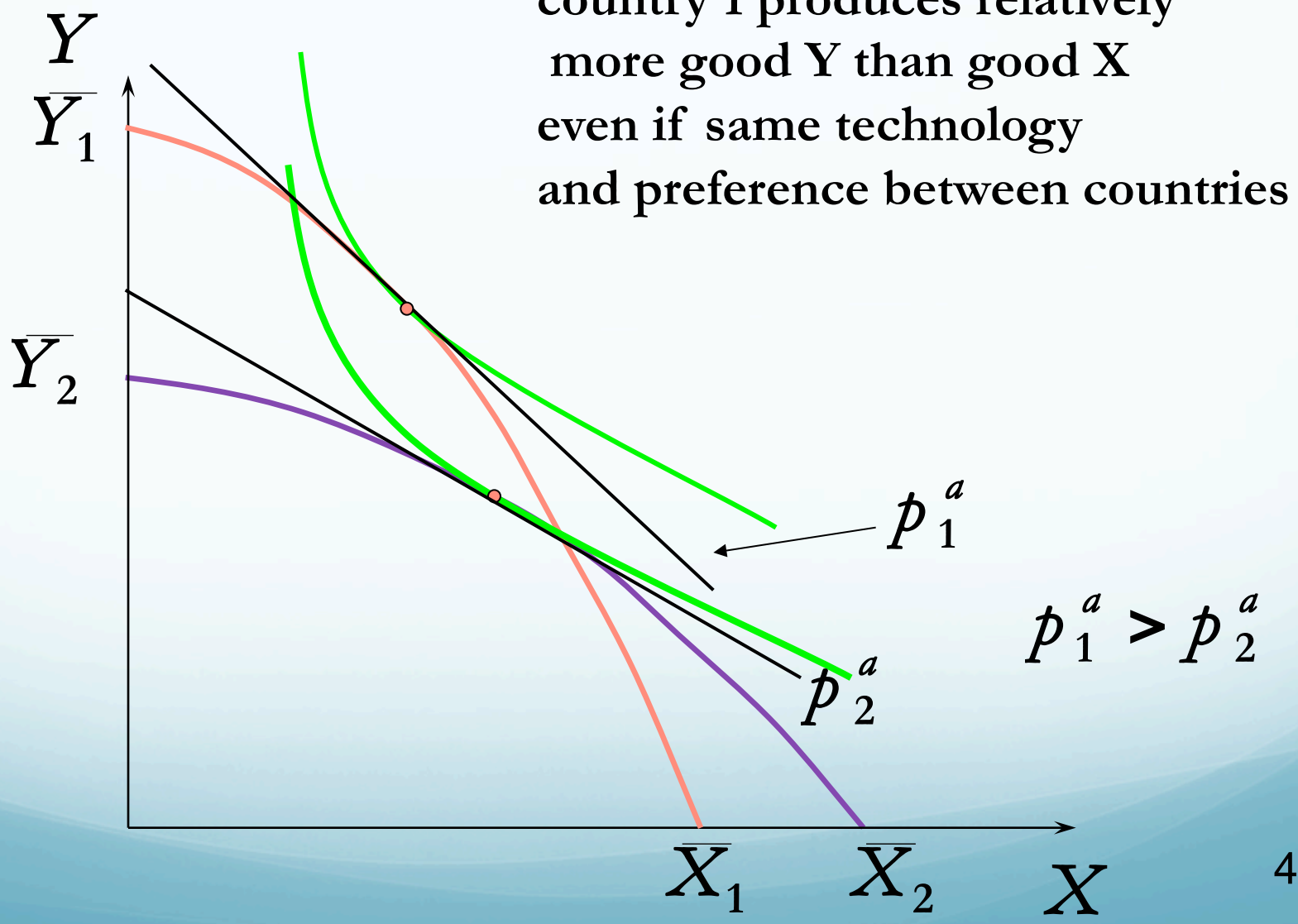
- **Production frontier**
  - **the production frontier expands in the direction of the good which is intensive in the country's relatively abundant factor**
  - **example: country 1 better endowed with capital**

## Figure: Production frontiers



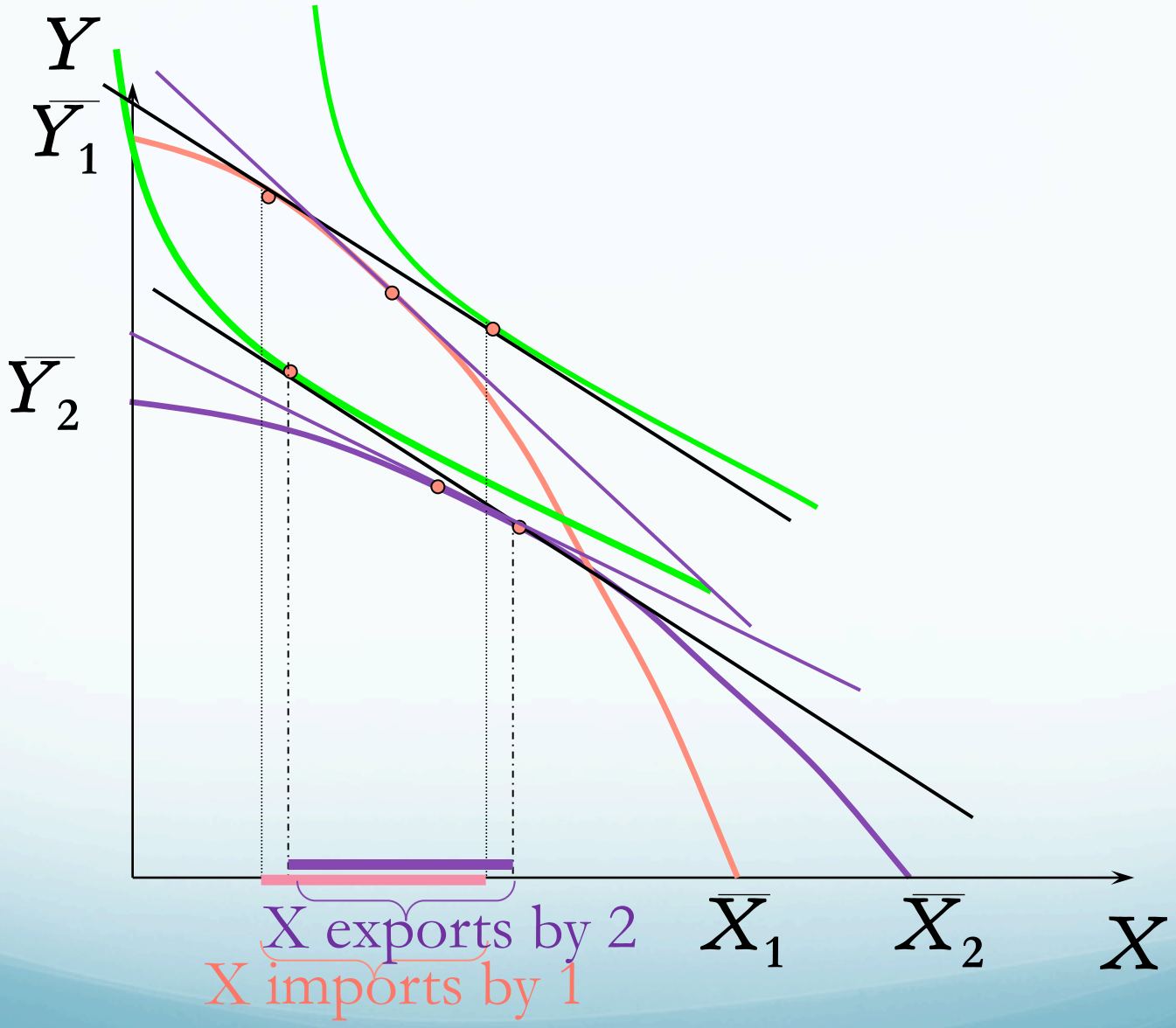
- General equilibrium under autarky
  - assumptions:
    - ✓ same technology across countries (standard assumption in neoclassical growth or trade models)
    - ✓ same preferences inside and across countries
  - graphic illustration: see next figure

## Figure: General equilibrium under autarky



- $p_1^a > p_2^a$ : the relatively scarce good is more expensive
- the equilibrium only depends on relative endowments
- $\Rightarrow$ the HO model is also called *factor-proportions* model

## Figure: General equilibrium under free trade



- **General equilibrium in open economy**
  - **perfect competition assumption is maintained**
  - **As seen earlier:**
    - ✓ **each country specializes incompletely**
    - ✓ **the world relative price is determined by excess demand equalization and lies between both autarky prices**

- **Heckscher-Ohlin theorem**

*A country exports the commodity that intensively uses its relatively abundant factor, and imports the other commodity.*

- **Note:**
  - **both countries export, even if a country has greater absolute endowments in both factors**
  - **reinterpretation : commodity trade is a substitute for factor trade**
- **Results depend on several assumptions:**
  - **perfect mobility of factors across sectors:**
    - ✓ **some adjustment costs could exist**
    - ✓ **see chapter IV and the specific-factor model**
  - **no international factor mobility: see next section**
  - **same homothetic preferences across countries: the theorem still applies if small differences / income effects, not if they are large**
  - **no trade distortions: taxes, transport costs…: see chapter VI**
  - **no factor intensity reversal, monotonic excess demands**

- **4. Other Effects of Trade Liberalization**
- **4.1 The Factor Price Equalization Theorem**
- *Free trade in commodities equalizes the factor price through the equalization of the relative commodity price, so long as both countries produce both goods (no complete specialization)*

- **Proof:**

  Because of perfect competition and constant returns to scale, prices are equal to marginal costs

$$p_y^1 = p_y^* = c_y(r^1, w^1) \qquad p_y^2 = p_y^* = c_y(r^2, w^2)$$

and

$$p_x^1 = p_x^* = c_x(r^1, w^1) \qquad p_x^2 = p_x^* = c_x(r^2, w^2)$$

  As cost functions are identical, the two sets of 'zero-profit conditions' are identical and have the same unique solution.

- Important implication:

  even if factors are immobile, their **real** returns are equalized across countries

  ⇒ even if factors were mobile, they would not move in equilibrium

  ⇒ "equivalence" between commodity mobility and factor mobility

  in this framework, **free movement of commodities mobility is a perfect substitute to free movement of production factors**

- Assumptions behind this result:
  - perfect free trade
  - perfect technology diffusion
  - perfect competition
  - perfect factor mobility across sectors

- A more general prediction would be:

  trade in commodities reduces international differences in factor returns

- **4.2 The Stolper-Samuelson Theorem**
- **Study of the factor price variations when trade is liberalized**

⇒ **implications in terms of political economy: first results on who agrees and disagrees on trade liberalization**

*A relative increase in the price of a commodity, increases the real return to the factor used intensively in that sector and reduces the real return to the other factor, so long as both goods continue to be produced.*

- Intuitions:
  - imagine that r and w do not change

    therefore, factor intensities are fixed in each sector

$$k_X = \frac{K_X}{L_X} < k_Y = \frac{K_Y}{L_Y}$$

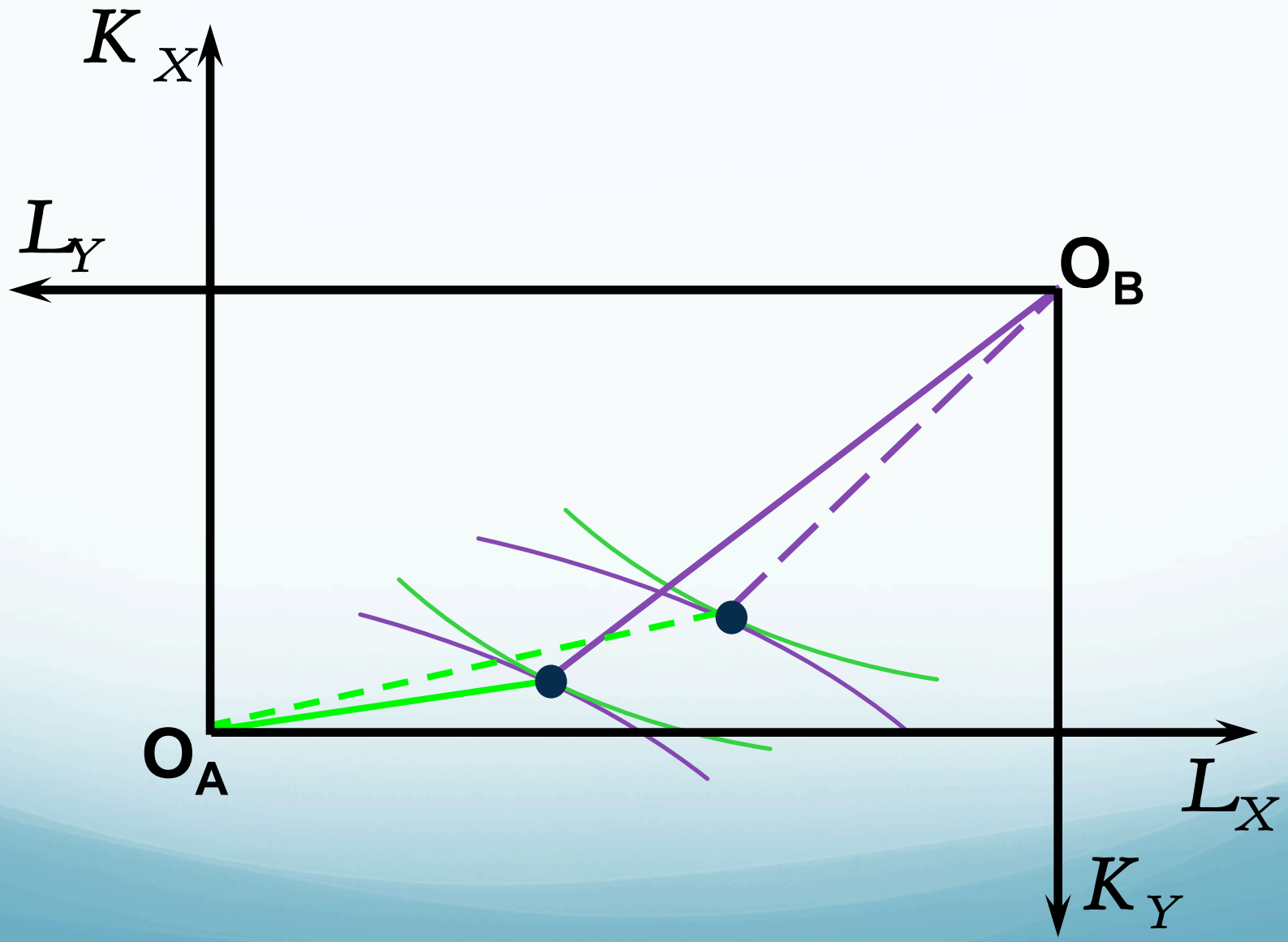
  - an increase in pX implies a reallocation of factors from Y to X

    Reallocating 1 unit of labor from Y to X frees $k_Y$ units of capital in sector Y and occupies $k_X$ units of capital in sector X.

    As $k_Y > k_X$, the reallocation of labor from Y to X generates an excess supply of capital: either w increases or r decreases.

  - Alternative intuition: in equilibrium, any reallocation of production from *Y* to *X* increases capital intensity in both sectors (convex efficient allocation curve) and therefore decreases the real return of capital and increases the real return to labor

- **Figure: Factor intensity variations in the labor abundant country**



- Then

$$\frac{\partial F_X}{\partial L_X}\left(\frac{K_X}{L_X}\right) = w \,/\, p_x \quad \text{and} \quad \frac{\partial F_Y}{\partial L_Y}\left(\frac{K_Y}{L_Y}\right) = w \,/\, p_Y$$

$$+ \qquad\qquad +$$

$$\frac{\partial F_X}{\partial K_X}\left(\frac{K_X}{L_X}\right) = r \,/\, p_x \quad \text{and} \quad \frac{\partial F_Y}{\partial K_Y}\left(\frac{K_Y}{L_Y}\right) = r \,/\, p_Y$$

$$- \qquad\qquad -$$

which proves that in the labour abundant country the real return of labour increases while the real return of capital decreases.

The opposite applies to the other country.

- Implications in terms of "political economy"
  - free trade induces specialization in the production of the good that uses more intensively the more abundant factor

    ⇒ the return to this factor increases, whereas the return to the other one decreases

    ⇒ if each consumer owns labor and capital in the same proportions as the country, she agrees on trade liberalization

⇒ if each consumer owns one type of factor:

- ✓ the relatively abundant factor owners favor trade liberalization since their real income increases
- ✓ the relatively scarce factor owners are against trade liberalization since their real income decreases

⇒ redistributive effect of trade in favor of the relatively abundant factor

- Relative, not absolute abundance matters
- factor owners who gain from free trade in one country lose in the other

  ⇒ no international agreement of labor owners (or of capital owners) on trade liberalization

  ⇒ illustration of the possible conflicts between individual and total surplus, between local and international surplus
- if non-costly lump-sum transfers exist, it is possible to make all factor owners better off in all countries
- taxes on imports introduce distortions and reduce trade gains but may reduce inequalities: see more in chapter VI

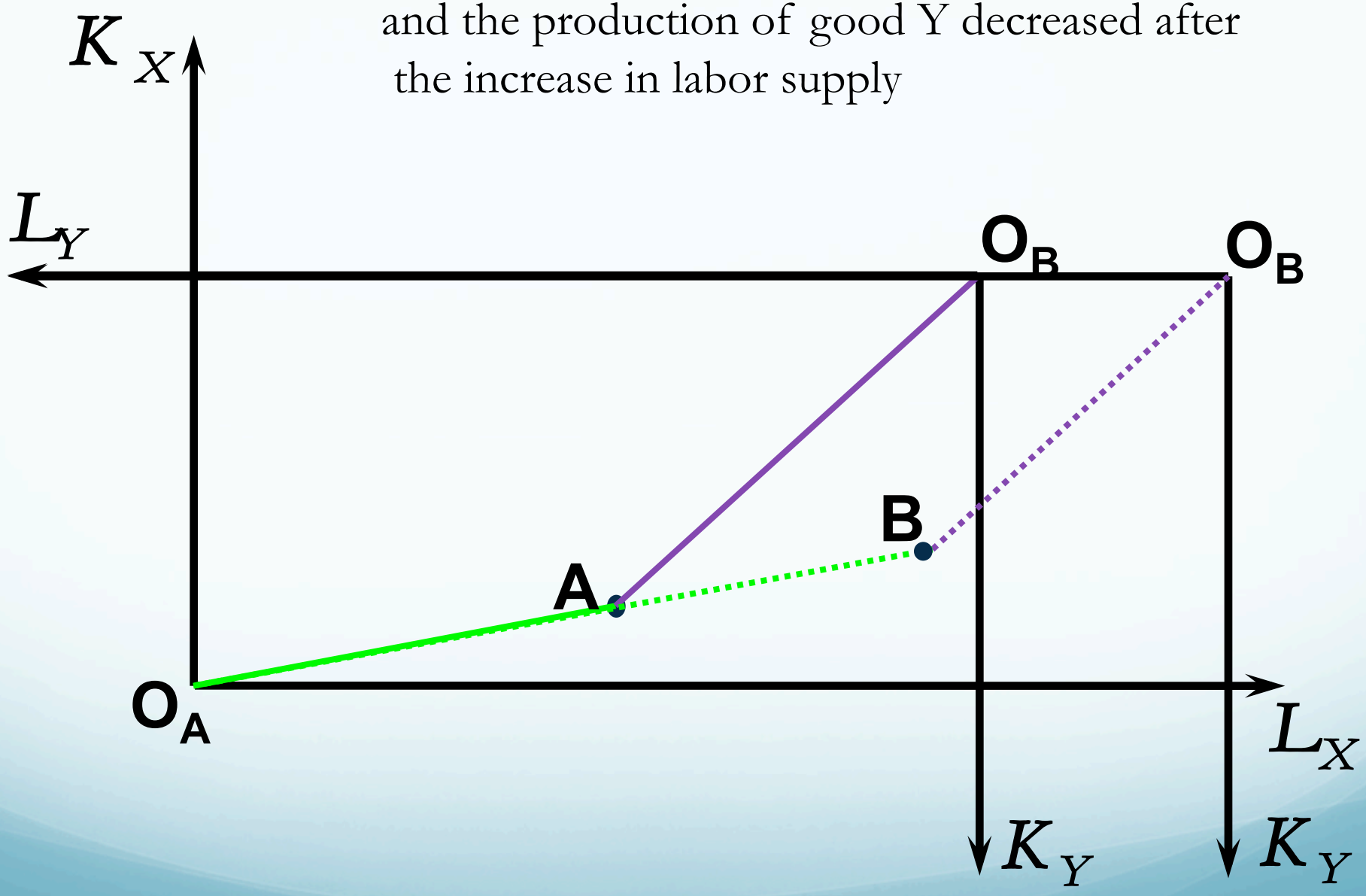
- **4.3 The Rybczynski Theorem**
- **Effects of a change in country endowments**
- **Small-country simplifying assumption: no impact of the variation on the world price**
- **Rybczynski theorem**

*If the relative price is constant and if both commodities continue to be produced, an increase in the supply of a factor leads to an increase in the output of the commodity using that factor intensively, and a decrease in the output of the other commodity.*

- Intuitions of the proof:
  - if $p$ remains constant, $w$ and $r$ also remain constant, as the **capital-labor ratio**, in both sectors

    ⇒ see next figure

The production of good X increased and the production of good Y decreased after the increase in labor supply



- $Kx$ / $Lx$ is constant and $Lx$ increases

  $\Rightarrow$ $Kx$ increases

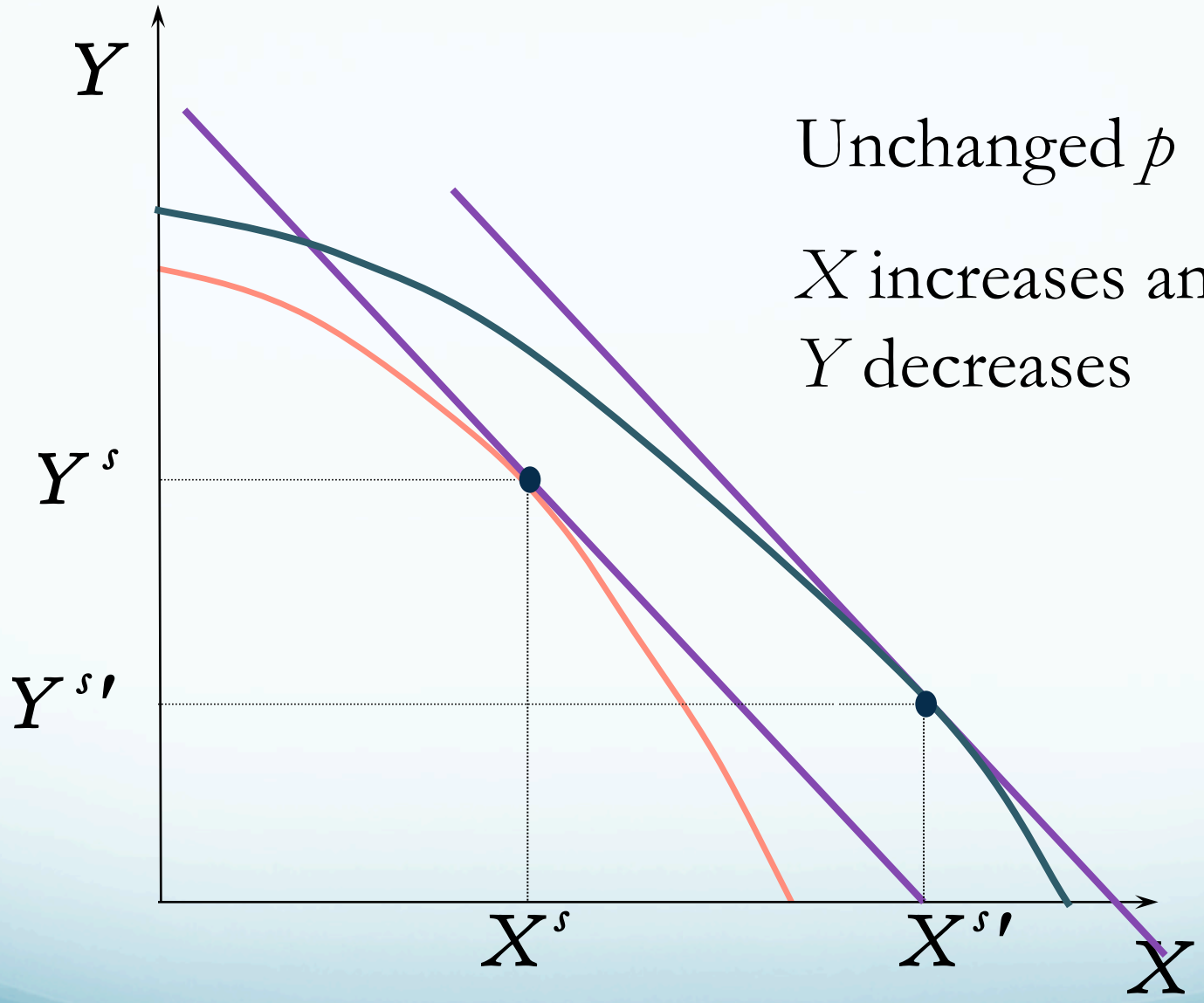
  but $\overline{K}$ is constant $\Rightarrow$ $Ky$ decreases

  now $Ky$ / $Ly$ is also constant

  $\Rightarrow$ $Ly$ decreases

  $Ly$ and $Ky$ decrease $\Rightarrow$ $Y$ decreases

- Another way to see this: see next figure


Y
Unchanged p
X increases and
Y decreases
Y^s
Y^s'
X^s
X^s'
X

- This theorem is at given $p$ and therefore is a partial equilibrium one.
- It is possible to show that the theorem holds when p is endogeneous.
- Differences in factor endowment growth may arise from differences in
  - savings behavior (eg differences in discount factor)
  - immigration policy
  - birth & mortality rates

- **4.4 Generalization of the theorems**

- The results can be generalized to any number of goods and factors under additional assumptions:
  - if there are more goods than factors, the theorems generalize under mild assumptions
    - Heckscher-Ohlin-Vanek: exports are more intensive in the country's abundant factors that imports
    - The theorem predicts the *factor content* of traded goods although exact trade patterns may be indeterminate.
    - Factor abundance is defined as a disproportionate share of the world endowment in that factor.
  - if there are more factors than goods factor prices are indeterminate in zero-profit conditions. But we can study the special case of the specific-factors model.

## References

Markusen, J., J. Melvin, W. Kaempfer, and K. Maskus, 1995. International Trade - Theory and Evidence, Mc Graw-Hill. Chapters 5 and 8.