Trade under Imperfect Competition and the Gravity Equation

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http://gregory.corcos.free.fr/ECO434/ECO434.html

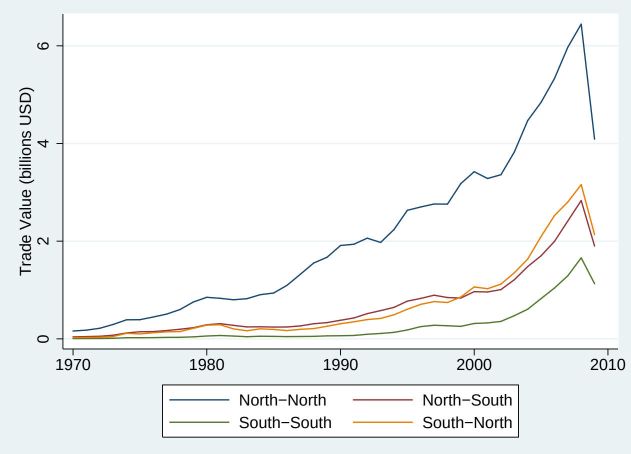
Outline

- **Introduction**: trade in similar products between similar countries
- **The Krugman Model**
  - Assumptions
  - Autarky and open-economy equilibria
- **Empirical Evidence: the Gravity Equation**
  - Microfoundations
  - Main lessons

Introduction

- Ricardo/HOS predict trade in **different** goods by **different** countries:
  - differences in productivity (Ricardo) or factor endowments (HOS)
  - specialization in comparative advantage goods
- These models do not explain
  - trade between similar countries
  - trade in similar goods
  - the 'gravity equation' (seen in Chapter 1)
- We need a new, imperfect competition trade model
  - similar countries trade differentiated varieties of the same goods
  - consumers gain from additional product diversity

World Trade by Region



Most of world trade occurs between *similar* countries. Source: UN ComTrade

Table: Intra-industry trade (IIT) shares in bilateral trade flows (top 10)

Top total IIT shares (per cent)		
Germany	France	86.20
Netherlands	Belgium and Luxembourg	85.01
France	Belgium and Luxembourg	80.42
France	United Kingdom	77.08
Germany	Switzerland	76.99
Germany	Belgium and Luxembourg	76.83
Austria	Germany	76.63
France	Spain	76.55
Germany	Netherlands	76.01
Canada	United States	73.55

Source: Fontagné, Freudenberg and Gaulier (2006). Intra-Industry Trade is defined as simultaneous trade of narrowly defined goods in both directions between two countries. This Table reports the 10 country pairs with the highest IIT shares.

The Krugman Model

Paul Krugman
(1953-)

Scale Economies, Product Differentiation and the Pattern of Trade, *American Economic Review*, 1980

Overview

- Ingredients:
    - Countries have identical endowments, technology and preferences.
    - Economies of scale
    - Differentiated (imperfectly substitutable) products.
    - Monopolistic competition: firms have market power on their variety.
    - Free entry: the number of firms is endogenous
- All the neoclassical motives for trade are absent!
- Trade occurs because consumers have demand for foreign varieties, and large trade partners offer more varieties.

Consumption

- One factor: labor (L). National income: wL.
- Utility defined over a continuum of **varieties** Ω:

$$\max_{q(\omega)} U = \max_{q(\omega)} \left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \text{ s.t. } \int_{\Omega} p(\omega) q(\omega) d\omega = wL$$

with $\sigma > 1$ the **elasticity of substitution** between varieties.

- Utility maximization yields the **demand function**:

$$q(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{wL}{P} \tag{1}$$

with $P = \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$

- P is the "ideal price index": 1 unit of utility costs P units of income.
- To see this, plug demand functions into the utility function

$$
\begin{aligned}
U &= \left(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} = \left(\int_\Omega \left(\frac{p(\omega)}{P}\right)^{-\sigma\frac{\sigma-1}{\sigma}} \left(\frac{wL}{P}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} \\
&= \frac{wL}{P} P^{\sigma} \left(\int_\Omega p(\omega)^{1-\sigma} d\omega\right)^{\frac{\sigma}{\sigma-1}} = \frac{wL}{P}
\end{aligned}
$$

- P is lower than the simple average of prices $p(\omega)$:
  $P = \left(\int_\Omega p(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}} < \int_\Omega p(\omega) d\omega$
- **Love of variety**: for a given nominal income and average price, increased product diversity increases welfare.

Production

- Each firm has a **monopoly** over one variety (ω), which is imperfectly substitutable with other varieties (monopolistic competition).
- **Production function**: to produce $q(\omega)$ firms need $f + \frac{q(\omega)}{\varphi}$ labor units
- The firm chooses a price $p(\omega)$, given w and P, to solve

$$\begin{cases} & \max_{p(\omega)} \left[p(\omega)q(\omega) - w\left(f + \frac{q(\omega)}{\varphi}\right)\right] \\ \text{s.t.} & q(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\sigma} \frac{wL}{P} \end{cases}$$

- **Optimal price**:

$$p = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi} \tag{2}$$

▸ Details on the optimal price

- **Free entry**:

$$\pi(\omega) = 0 \Rightarrow q(\omega) = (\sigma - 1)\varphi f \tag{3}$$

- **Labor market equilibrium**

$$n\left(f + \frac{q}{\varphi}\right) = L \Rightarrow n = \frac{L}{\sigma f} \tag{4}$$

⇒ The number of firms increases with market size (L) and decreases with fixed costs (f) and competition (σ).

- (1), (2), (3), (4) jointly define the autarky equilibrium.

Open Economy Equilibrium

- Consider two identical countries except for their size: L, L^*.
- 'Iceberg' transport costs: $\tau \geq 1$ units must be shipped to export 1 unit. $\tau - 1$ represents the transport cost.
- Producers solve

$$\begin{cases} \max_{p^D(\omega), p^X(\omega)} \left[p^D(\omega) q^D(\omega) + p^X(\omega) q^X(\omega) - w \left(f + \frac{q^D(\omega) + \tau q^X(\omega)}{\varphi} \right) \right] \\ \text{s.c.} \quad q^D(\omega) = \left(\frac{p^D(\omega)}{P} \right)^{-\sigma} \frac{wL}{P} \\ \qquad q^X(\omega) = \left(\frac{p^X(\omega)}{P^*} \right)^{-\sigma} \frac{w^* L^*}{P^*} \end{cases}$$

- Optimal prices $p^D = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}$ and $p^X = \tau \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}$

- Total production: $q \equiv q^D + \tau q^X$
- Total profit:
  $\pi = (p^D - \frac{w}{\varphi})q^D + (\tau p^D - \tau \frac{w}{\varphi})q^X - wf = \frac{w}{(\sigma-1)\varphi} q - wf$
- Free entry:

$$\pi = 0 \Rightarrow q = (\sigma - 1)\varphi f$$

- Labor market equilibrium:

$$n\left(f + \frac{q}{\varphi}\right) = L \Rightarrow n = \frac{L}{\sigma f}, n^* = \frac{L^*}{\sigma f}$$

- Trade balance

$$np^X q^X = n^* p^{X*} q^{X*}$$

Gains From Trade

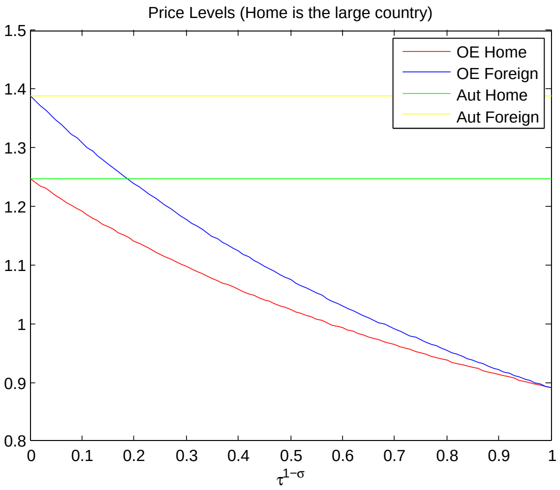
- Greater variety reduces the ideal price index, at given wages.

$$
\begin{aligned}
P &= \left(\int_0^n p^D(\omega)^{1-\sigma} d\omega + \int_0^{n^*} p^{X*}(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \\
&= \left(n \left(p^D \right)^{1-\sigma} + n^* \left(\tau p^{D*} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
&\leq P^a
\end{aligned}
$$

- $U = \frac{wL}{P}$ so at given wages a lower P implies gains from trade.
- When $\tau = 1$ prices are equal in both countries. When $\tau > 1$ prices are lower in the large country: fewer varieties bear shipping costs.

▸ Price indices, wages

Gains From Trade



Wages

- Substituting for prices, quantities and numbers of firms in the trade balance equation $np^X q^X = n^* p^{X*} q^{X*}$ yields

$$\frac{w}{w^*} = \left(\frac{P}{P^*}\right)^{\frac{1-\sigma}{\sigma}} = \left(\frac{Lw^{1-\sigma} + L^*\left(\tau w^*\right)^{1-\sigma}}{L\left(\tau w\right)^{1-\sigma} + L^* w^{*\,1-\sigma}}\right)^{\frac{1}{\sigma}}$$

  - When $\tau = 1$ wages are equal in both countries.
  - When $\tau \to +\infty$, $\frac{w}{w^*} \to \left(\frac{L}{L^*}\right)^{\frac{1}{2\sigma-1}}$. Thanks to better market access the large country's firms sell and hire disproportionately more.
- An extended model with mobile workers can explain economic agglomeration in some countries or regions.

Gravity Regressions

From Theory to Gravity Regressions

- In the model aggregate exports equal $X = np^X q^X$
- We predict that exports from i to j equal

$$X_{ij} = n_i \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{\varphi_i P_j} \right)^{1-\sigma} R_j$$

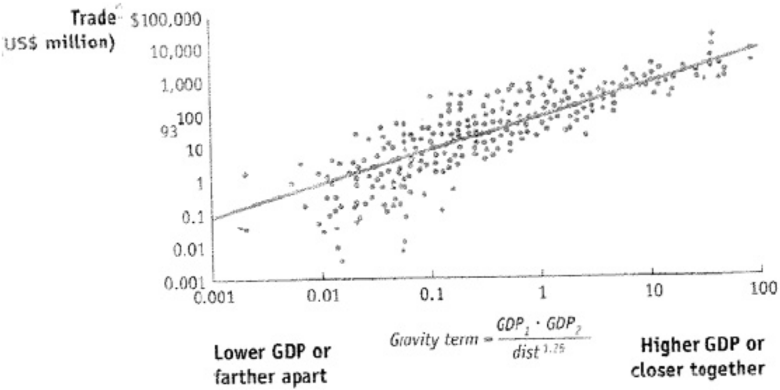
or in logs:

$$\ln X_{ij} = \underbrace{\ln \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma}}_{constant} + \underbrace{\ln n_i + (1-\sigma) \ln \frac{w_i}{\varphi_i}}_{i-specific} + \underbrace{\ln P_j^{\sigma-1} + \ln R_j}_{j-specific} + \underbrace{(1-\sigma) \ln \tau_{ij}}_{\text{transport cost}}$$

- 'Gravity' empirical equation

$$\ln X_{ij} = \kappa + \alpha \mathbf{X}_i + \beta \mathbf{M}_j + \gamma \ln dist_{ij} + \delta \mathbf{C}_{ij} + \varepsilon_{ij}$$

Trade between US States and Canadian Provinces



Source: Feenstra & Taylor (2011)

Gravity Equation Estimates

	Variable dependante: ln X_{ij}					
	(1)	(2)	(3)	(4)	(5)	(6)
ln Population i	0.799^a			1.185^a		
ln GDP/capita i	1.072^a			1.272^a		
ln Population j	0.723^a			0.896^a		
ln GDP/capita j	1.058^a			0.920^a		
ln Distance	-1.008^a			-1.511^a		
Trade Agreement						
GATT/WTO						
Common Currency						
Common Border						
Common Language						
Colonial Past						
Year	1970			2006		
Country Fixed Effects	No			No		
# Observations	9,035			16,936		
R^2	0.583			0.631		

Gravity Equation Estimates

	Dependent variable: ln X_{ij}					
	(1)	(2)	(3)	(4)	(5)	(6)
ln Population i	0.799[a]	0.823[a]		1.185[a]	1.191[a]	
ln GDP/capita i	1.072[a]	1.110[a]		1.272[a]	1.265[a]	
ln Population j	0.723[a]	0.740[a]		0.896[a]	0.900[a]	
ln GDP/capita j	1.058[a]	1.092[a]		0.920[a]	0.912[a]	
ln Distance	-1.008[a]	-0.838[a]	-1.000[a]	-1.511[a]	-1.199[a]	-1.619[a]
Trade Agreement		0.917[a]	0.643[a]		0.758[a]	0.493[a]
GATT/WTO		-0.011	0.038		0.306[a]	0.811[a]
Common Currency		1.470[a]	1.460[a]		-0.029	0.035
Common Border		0.588[a]	0.533[a]		1.152[a]	0.840[a]
Common Language		0.559[a]	0.535[a]		1.108[a]	0.909[a]
Colonial Past		1.376[a]	1.277[a]		0.672[a]	0.889[a]
Year	1970	1970	1970	2006	2006	2006
Country Fixed Effects	No	No	Yes	No	No	Yes
# Observations	9,035	9,035	9,035	16,936	16,936	16,936
R^2	0.583	0.607	0.710	0.631	0.649	0.741

The Distance Elasticity Over Time

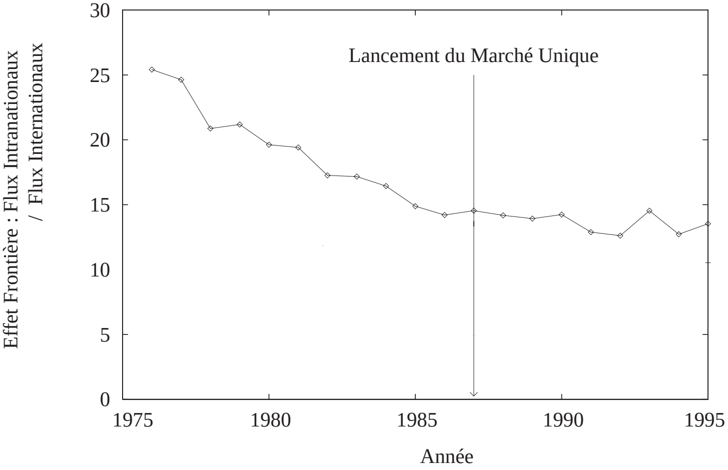
- An elasticity of 1 means that doubling distance reduces trade by half.
- Distance proxies transportation costs, time as a trade barrier, cultural distance and informational frictions.
- Increasing spatial concentration of trade over time...

Other Results

- **Trade Agreements**
  - Trade is 40% higher between partners of a trade agreement.
- **Currency Unions**
  - Trade is twice as high between members of a currency union.
- **Cultural Links**
  - Trade is higher between countries that share a language or historical links

Note: these are correlations. Stating causality requires further work...

The Border Effect in the EU



Source: Head & Mayer (2000)

Limits of the Gravity Equation

- The within-EU border effect remains an empirical puzzle.
- 'Zeroes':
  - trade flows are zero for over 50% of country pairs, even more when using sector-level trade flows
  - zeroes require special econometric techniques to estimate the gravity equation
  - zeroes require an extension of the Krugman model, where there is always some demand for each variety

Conclusions

- Neoclassical trade models explain trade in different goods between different countries.
- The Krugman trade model explain trade in similar goods between similar countries.
  - comparative advantage trade motives are absent
  - trade occurs because of demand for different varieties of the same product
  - product diversity creates gains from trade
- The model is consistent with 'gravity' regression results: trade flows are higher between countries that are large, close, share the same language, history, currency or have signed trade agreements.

Appendix

How to derive the demand function

- Lagrangian: $L = \left(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} - \mu\left(\int_\Omega p(\omega)q(\omega)d\omega - wL\right)$
- First order conditions:

$$\begin{aligned} \frac{\partial L}{\partial q(\omega)} &= q(\omega)^{\frac{-1}{\sigma}}\left(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{1}{\sigma-1}} - \mu p(\omega) = 0 \\ \Leftrightarrow \quad & q(\omega)^{\frac{-1}{\sigma}} U^{\frac{1}{\sigma}} = \mu p(\omega) \\ \Leftrightarrow \quad & q(\omega) = U\mu^{-\sigma} p(\omega)^{-\sigma} \end{aligned}$$

- Budget constraint: $wL = \int_\Omega p(\omega)q(\omega)d\omega = U\mu^{-\sigma}\int_\Omega p(\omega)^{1-\sigma}d\omega$
- Define the price index $P = \left(\int_\Omega p(\omega)^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}}$. The budget constraint is rewritten as

$$wL = U\mu^{-\sigma}P^{1-\sigma}$$

- Note that $wL = PU$. P is the monetary value of one unit of utility.

- From the budget constraint $wL = U\mu^{-\sigma}P^{1-\sigma}$ and the f.o.c. $q(\omega) = U\mu^{-\sigma}p(\omega)^{-\sigma}$, one obtains the demand function:

$$q(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\sigma} \frac{wL}{P}$$

- Demand for variety ω increases with overall purchasing power wL/P, and decreases with the relative price of variety ω.
- σ is equal to
    - the price elasticity: a 1% rise in $p(\omega)$ reduces demand by σ%
    - the elasticity of substitution: since $\frac{q(\omega)}{q(\omega')} = \left(\frac{p(\omega)}{p(\omega')}\right)^{-\sigma}$, increasing the relative price of the ω variety by 1% reduces the relative demand for this variety by σ%

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How to derive the optimal price

- Start from the firm's profit function:

$$\pi(\omega) = p(\omega)q(\omega) - w\left(f + \frac{q(\omega)}{\varphi}\right)$$

- Maximize with respect to price given demand and price index P

$\Rightarrow$ First order condition:

$$\frac{\partial\pi(\omega)}{\partial p(\omega)} = P^{\sigma-1}wL\left[(1-\sigma)p^{-\sigma} + \frac{w}{\varphi}\sigma p^{-\sigma-1}\right] = 0$$

Or after rearranging:

$$p = \underbrace{\frac{\sigma}{\sigma-1}}_{Mark-up}\ \underbrace{\frac{w}{\varphi}}_{Marginal\ cost}$$

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Price indices in a two-country world economy

- The price index is now written as:

$$P = \left(\sum_{\omega \in H} p(\omega)^{1-\sigma} + \sum_{\omega \in F} (\tau p^*(\omega))^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

- In the symmetric equilibrium, $p(\omega) = p$, $\forall \omega \in H$ and $p^*(\omega) = p^*$, $\forall \omega \in F$

$$\Rightarrow \quad P = \left(np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

$$and \quad P^* = \left(n(\tau p)^{1-\sigma} + n^* p^{*\,1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

- Absent transportation costs ($\tau = 1$), with identical countries, the two indices are equal: $P = P^* = (2n)^{\frac{1}{1-\sigma}} p$. Both countries have access to the same varieties in the same conditions.
- Both indices are lower than those in autarky, which are: $P = n^{\frac{1}{1-\sigma}} p$ and $P^* = n^{\frac{1}{1-\sigma}} p^*$
- At given wages, opening up the economy has a positive impact on welfare ($U = wL/P$). This comes from consumers' **preference for diversity**

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Wages

- We have expressed optimal prices $p(\omega)$ and P as functions of nominal income wL.
- L is exogenous but w is endogenous
- The wage is determined by the goods market equilibrium equation. Due to Walras' law, it is equivalent to rely on (i) the domestic market ($wL = \sum_\omega wl(\omega)$); (ii) the foreign market ($w^* L^* = \sum_\omega w^* l^*(\omega)$); (iii) the trade balance ($X = X^*$)
- We use the trade balance:

$$\lambda \times L \times L^* \times \left(\frac{\tau w}{P^*}\right)^{1-\sigma} \times w^* = \lambda \times L \times L^* \times \left(\frac{\tau w^*}{P}\right)^{1-\sigma} \times w$$

$$\Rightarrow \quad \frac{w}{w^*} = \left(\frac{P}{P^*}\right)^{\frac{1-\sigma}{\sigma}}$$

$$\textit{with} \quad \left(\frac{P}{P^*}\right) = \frac{np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma}}{n(\tau p)^{1-\sigma} + n^* p^{*\,1-\sigma}}$$

$$\Rightarrow \quad \frac{w}{w^*} = \left(\frac{Lw^{1-\sigma} + L^*(\tau w^*)^{1-\sigma}}{L(\tau w)^{1-\sigma} + L^* w^{*\,1-\sigma}}\right)^{1/\sigma}$$

Relative imports:

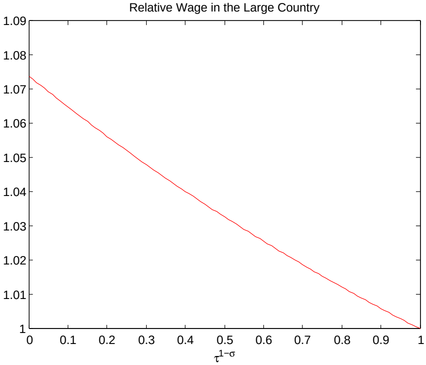
$$\frac{M}{M^*} = \frac{n^*}{n}\left(\frac{\tau p^*/P}{\tau p/P^*}\right)^{1-\sigma}\frac{wL}{w^*L^*} = \frac{w}{w^*}\left(\frac{w^*/P}{w/P^*}\right)^{1-\sigma}$$

- Starting from the symetric equilibrium: $\frac{L}{L^*} = \frac{w}{w^*} = \frac{M}{M^*}$
- An increase in L/L^* increases the relative number of firms in H which reduces P/P^*. This makes foreign goods relatively more expansive $\rightarrow \downarrow M/M^*$
- For trade to be balanced, needs to be compensated by an increase in the relative wage $w/w^* \rightarrow \uparrow M/M^*$ through an income effect ($\uparrow$ aggregate demand) and a substitution effect ($\downarrow$ relative competitiveness of domestically produced varieties)

$\Rightarrow$ Wages are relatively high in large countries Back to section 1

Wages (2)

Relative wage in the large country, as a function of the "freeness" of trade $\tau^{1-\sigma}$



Welfare gains

- Autarky: $P = pn^{\frac{1}{1-\sigma}}$ and $P^* = p^* {n^*}^{\frac{1}{1-\sigma}}$
- Open economy: $P = \left[p^{1-\sigma} n + (\tau p^*)^{1-\sigma} n^*\right]^{\frac{1}{1-\sigma}}$ and $P^* = \left[{p^*}^{1-\sigma} n^* + (\tau p)^{1-\sigma} n\right]^{\frac{1}{1-\sigma}}$
- Without transportation costs: $P = P^* = (2np^{1-\sigma})^{\frac{1}{1-\sigma}} < (np^{1-\sigma})^{\frac{1}{1-\sigma}}$ since $\sigma > 1$

$\Rightarrow$ **Opening up the economy yields a welfare gain deriving from more diversity.**

- In Krugman (1979), trade has a pro-competitive effect too (fall in p due to a rise in σ).

Prices as a function of the "freeness" of trade $\tau^{1-\sigma}$